

Exponential Functions

Ontario MCR3U • exponent laws, rational exponents, graphs, transformations, growth and decay

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Worksheet, answers &
this cheat sheet



MOST TESTED A car worth \$32 000 loses 15% of its value each year. What is it worth after 4 years? (card 8)

Linear, quadratic or exponential?

Constant first
differences

Linear

Constant
second
differences

Quadratic

Constant
ratio

Exponential

1 Exponent laws

$$a^m a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn} \quad (ab)^n = a^n b^n$$

USE Keep the base; never multiply bases. A power applies to every factor in the bracket, coefficients included.

TRAP $4^2 \cdot 4^3 = 4^5$, not 16^5 : add the exponents, keep the base.

TRAP $(-2x^2)^4 = 16x^8$: an even power of a negative is positive.

2 Zero and negative exponents

$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n}$$

USE A negative exponent means a reciprocal, not a negative number. It applies only to its own base.

TRAP $3x^{-2} = 3/x^2$, not $1/(3x^2)$: the -2 belongs to x only.

3 Rational exponents

$$a^{1/n} = \sqrt[n]{a} \quad a^{m/n} = (\sqrt[n]{a})^m$$

USE Take the root first, then the power: the numbers stay small. The same laws apply: $x^{1/2} x^{1/3} = x^{5/6}$.

EXAMPLE $8^{-2/3} = 1/(\sqrt[3]{8})^2 = 1/4$.

TRAP $x^{2/3} = 9$ gives $x = \pm 9^{3/2} = \pm 27$: an even numerator allows two answers.

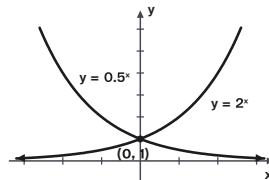
4 Scientific notation

USE $a \times 10^n$ with $1 \leq a < 10$. Multiply the a 's, add the exponents, then put a back between 1 and 10.

TRAP $(3.0 \times 10^{-4})(5.0 \times 10^2) = 15 \times 10^{-2} = 1.5 \times 10^{-1}$: a smaller a raises the exponent.

5 The graph of $y = b^x$

USE $b > 0$, $b \neq 1$. Domain $\mathbb{R}$, range $y > 0$, y -intercept 1, asymptote $y = 0$. $b > 1$ grows; $0 < b < 1$ decays.



TRAP $y = -5^x$ means $-(5^x)$: at $x = 2$ it is -25 , not 25 .

6 Which model fits?

USE Equal x -steps: constant differences mean linear; constant second differences, quadratic; a constant ratio y_{n+1}/y_n , exponential. Check **every** difference.

EXAMPLE 5, 15, 45, 135: ratio 3, so $y = 5(3)^x$ if the table starts at $x = 0$.

TRAP If the table starts at $x = 1$, a is not the first y -value: a is the value at $x = 0$.

7 Transformations

$$y = a b^{k(x-d)} + c$$

USE Factor k out of the exponent first. Map $(x, y) \rightarrow (x/k + d, ay + c)$. The asymptote moves to $y = c$.

TRAP $5^{3x-6} = 5^{3(x-2)}$: right 2, not 6.

8 Growth and decay

$$A = A_0(1+r)^t \quad A = A_0(1-r)^t$$

$$A = A_0(2)^{t/d} \quad A = A_0\left(\frac{1}{2}\right)^{t/h}$$

USE d is the doubling time and h the half-life, in the same units as t .

EXAMPLE \$32 000 losing 15% a year: $32\,000(0.85)^4 \approx \$16\,704.20$ after 4 years.

TRAP Losing 12% a year: the base is $1 - 0.12 = 0.88$, not 0.12.

TRAP Doubling every 8 h: the exponent is $t/8$, not $8t$.