

Exponential Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 50 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Exponent laws: $a^m \cdot a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$. The base stays the same: never multiply the bases.
 $y = b^x$ ($b > 0$, $b \neq 1$): domain $(-\infty, \infty)$, range $(0, \infty)$, y -intercept 1, asymptote $y = 0$. Growth if $b > 1$, decay if $0 < b < 1$.

Models: $A = A_0(1+r)^t$, $A = A_0(1-r)^t$, $A = A_0\left(\frac{1}{2}\right)^{t/h}$. Linear data has constant differences; exponential data has constant ratios.

Show your work on separate paper.

Core Skills (K)

Integer Exponents · Rational Exponents · Simplifying · Properties of b^x · Transformations

K1 Integer Exponents

Write each answer as a fraction or with positive exponents.

- Evaluate $2^{-3} + 3^0$.
- Evaluate $\left(\frac{2}{5}\right)^{-2}$.
- Simplify $\frac{(-2x^3)^4}{8x^{-2}}$.

K2 Rational Exponents

Evaluate without a calculator.

- $27^{2/3}$
- $16^{-3/4}$
- $\left(\frac{4}{9}\right)^{-3/2}$

K3 Simplifying

Simplify. Write each answer with positive exponents.

- $x^{1/2} \cdot x^{1/3}$
- $\frac{a^{3/4}b^{-2}}{a^{-1/4}b}$
- $(8x^6y^{-3})^{-2/3}$

K4 Properties of $y = b^x$

- Determine which of these are exponential functions:
 $y = x^4$, $y = 4^x$, $y = 1^x$, $y = (-4)^x$, $y = 0.4^x$.
- For $f(x) = 2^{-x}$, state the domain and range in interval notation, the y -intercept and the equation of the horizontal asymptote.
- Determine whether $f(x) = 2^{-x}$ is increasing or decreasing, and evaluate $f(-3)$.

$a^0 = 1$, $a^{-n} = \frac{1}{a^n}$, $a^{m/n} = (\sqrt[n]{a})^m$. A negative exponent means a reciprocal, **not** a negative number.

$y = a \cdot b^{k(x-d)} + c$: factor k out of the exponent before reading the shift d . The asymptote moves to $y = c$.

K5 Transformations

Let $g(x) = 2 \cdot 3^{2x-4} + 1$.

- Write g in the form $y = a \cdot b^{k(x-d)} + c$ and state a , k , d and c .
- State the equation of the horizontal asymptote and the range of g .
- The point $(1, 3)$ is on $y = 3^x$. Determine its image on the graph of g .

Real World (A)

Half-Life · Doubling · Linear vs Exponential

A1 Medical Tracer

A hospital in Hamilton prepares a dose of a medical tracer with an activity of 800 MBq. The tracer has a half-life of 6 hours.

- Write an equation for the activity A , in MBq, after t hours. Then find the activity one day after preparation.
- A scan needs an activity of at least 25 MBq. For how many hours after preparation can the dose be used?
- According to the model, when is the tracer completely gone? Does this match what happens to a real dose?

A2 Petri Dish

A lab culture starts with 500 bacteria at 9:00 a.m. and doubles every 20 minutes.

- Write an equation for the number of bacteria N after t minutes.
- How many bacteria are there at 11:00 a.m.?
- At what time does the culture reach 256 000 bacteria?
- Evaluate $N(1440)$, the model's count after one day, in scientific notation. Each bacterium has a mass of about 1×10^{-12} g. Can the dish hold this many bacteria?

A3 Two Towns

The table shows the populations of two Ontario towns t years after 2020.

t	0	1	2	3
Town A	12 000	12 600	13 200	13 800
Town B	8 000	8 800	9 680	10 648

- Determine which town grows linearly and which grows exponentially. Write an equation for each population.
- In which year does Town B first have more people than Town A?
- Use each model to estimate the population in 1995 ($t = -25$). Which estimate is impossible, and what does that tell you about that model?

Think It Through (T)

Error Analysis · Building Functions

T1 Spot the Flaw

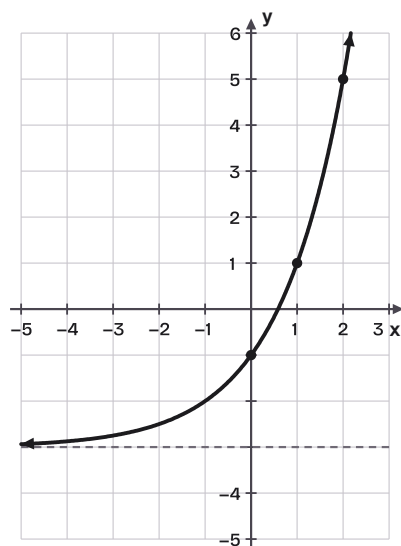
A student solves $x^{2/3} = 16$:

"Raise both sides to the power $\frac{3}{2}$: $x = 16^{3/2} = 64$. So $x = 64$ is the only solution."

- Evaluate $(-64)^{2/3}$. What does the result tell you about the student's claim?
- Identify the step where the student lost a solution, and give both solutions.
- Solve $x^{2/5} = 4$ and $x^{3/4} = -8$.

T2 Build It Backwards

- The graph shows a function of the form $y = a \cdot 2^x + c$, with its asymptote dashed. Each grid square is 1 unit. Determine a and c .



- Write an exponential function with range $(-\infty, 4)$ and y -intercept 1. Many answers are possible.

- Write a decreasing exponential function with asymptote $y = 2$ whose graph passes through $(1, 5)$. Many answers are possible.

Boss Level

Optional challenge

Hidden Quadratics

Solve over the real numbers.

- $4^x - 3 \cdot 2^{x+1} + 8 = 0$
- $9^x - 3^{x+1} = 54$
- Find every value of k for which $4^x - k \cdot 2^x + 9 = 0$ has exactly one real solution.



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