

Introduction to Functions

Ontario MCR3U • functions, notation, parent functions, domain and range, inverses

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Worksheet, answers &
this cheat sheet



MOST TESTED Find $f^{-1}(x)$ for $f(x) = \frac{2x+1}{5}$, then state the domain and range of f^{-1} . (card 7)

1 Function or relation?

USE A **function** gives each input exactly one output. Graph: no vertical line meets it twice. Mapping: no input has two arrows. Outputs may repeat.

EXAMPLE $\{(1, 4), (2, 4), (3, 7)\}$ is a function; $\{(4, 1), (4, 2)\}$ is not.

TRAP Test **every** vertical line. $x = y^2$ passes at $x = -1$ but fails at $x = 4$: $y = \pm 2$.

2 Evaluating f

USE Replace **every** x with the input, in brackets. $f(x)$ is the output at x , not f times x .

EXAMPLE $f(x) = x^2 - 3x$:
 $f(-2) = (-2)^2 - 3(-2) = 10$;
 $f(2a) = 4a^2 - 6a$.

TRAP For $f(x) = x^2$,
 $f(-3) = (-3)^2 = 9$, not $-3^2 = -9$:
bracket a negative input.

TRAP $f(x+1) \neq f(x) + f(1)$. For
 $f(x) = x^2$, $f(x+1) = x^2 + 2x + 1$.

3 Solving $f(x) = k$

USE $f(a)$: you know the input, find the output. $f(x) = k$: you know the output, solve for the input(s). x -intercepts: solve $f(x) = 0$.

EXAMPLE $f(x) = x^2 - 3x$, $f(x) = 4$:
 $x^2 - 3x - 4 = 0$, so $x = 4$ or $x = -1$.

TRAP $T(e) = 0$ asks for the elevation where it is 0°C ; $T(0)$ is the temperature at sea level.

4 Parent functions

$f(x)$	domain	range
x	$\mathbb{R}$	$\mathbb{R}$
x^2	$\mathbb{R}$	$y \geq 0$
$\sqrt{x}$	$x \geq 0$	$y \geq 0$
$ x $	$\mathbb{R}$	$y \geq 0$
$\frac{1}{x}$	$x \neq 0$	$y \neq 0$

TIP $y = x^2$ and $y = |x|$ share a vertex, domain and range, but one is a U and the other a V. $y = 1/x$ has asymptotes $x = 0$ and $y = 0$.

TRAP $\sqrt{(-5)^2} = \sqrt{25} = 5$, not -5 : a square root never outputs a negative.

5 Finding the domain

USE Polynomial: all real numbers. Square root: radicand ≥ 0 . Rational: denominator $\neq 0$, found **before** simplifying.

EXAMPLE $f(x) = \sqrt{10-2x}$:
 $-2x \geq -10$, so $x \leq 5$: domain $(-\infty, 5]$.

TRAP Dividing by a negative flips the sign: $-2x \geq -10$ gives $x \leq 5$, not $x \geq 5$.

TRAP $(x-2)/(x^2-4)$ excludes $x = 2$ and $x = -2$, even though $x-2$ cancels.

6 Range and notation

$$\{x \in \mathbb{R} \mid x \geq 3\} = [3, \infty)$$

USE Square bracket: included. Round: excluded. $\pm\infty$ always round. Range from a graph: read the y -values, skip gaps: $[-2, 1] \cup [3, 5]$. In context, keep only values that make sense.

TRAP A thrown ball's domain runs until it lands ($t = 4$), not until its peak ($t = 2$).

7 The inverse f^{-1}

$$f(f^{-1}(x)) = x \quad f^{-1}(f(x)) = x$$

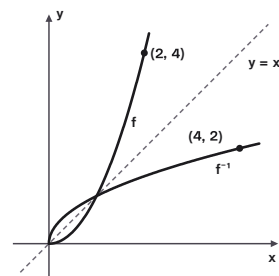
USE Swap x and y , then solve for y . Domain of f^{-1} = range of f , and range of f^{-1} = domain of f .

EXAMPLE $f(x) = (2x+1)/5$:
 $x = (2y+1)/5$, so $f^{-1}(x) = (5x-1)/2$.

TRAP $f^{-1}(x)$ is the inverse function, not $1/f(x)$.

8 Graph of the inverse

USE Reflect over $y = x$: (a, b) becomes (b, a) . If f fails the horizontal-line test, restrict its domain so f^{-1} is a function.



EXAMPLE $f(x) = x^2$, $x \geq 0$:
 $f^{-1}(x) = \sqrt{x}$.

TRAP Reflect over the line $y = x$, never over an axis: swap the coordinates.