

# Quadratic Functions

Ontario MCR3U · forms, max/min, radicals, inverses, solving, linear-quadratic systems

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Worksheet, answers &  
this cheat sheet



**MOST TESTED** Write  $y = -2x^2 + 8x + 3$  in vertex form and state the maximum value. (card 2)

## Which method?

It factors with small integers

**Factor**

Need the vertex

**Complete the square**

It doesn't factor nicely

**Quadratic formula**

### 1 Three forms

$$y = a(x - h)^2 + k: \text{vertex } (h, k)$$

$$y = a(x - r)(x - s): \text{zeros } r, s$$

$$\text{axis of symmetry } x = (r + s)/2$$

$$y = ax^2 + bx + c: \text{y-intercept } c$$

**USE**  $a > 0$  opens up;  $|a| > 1$  is narrower than  $y = x^2$ .

**TRAP**  $y = (x - 2)(x + 6)$  has zeros 2 and -6, so the axis is  $x = -2$ , not  $x = 2$ .

**TRAP**  $3(x - 2)^2 \neq (3x - 6)^2$ : square first, then multiply by 3.

### 2 Completing the square

**USE** Factor  $a$  from the  $x^2$  and  $x$  terms. Add and subtract  $(b/2)^2$  inside. The subtracted number is multiplied by  $a$  as it comes out.

#### EXAMPLE

$$-2x^2 + 8x + 3 = -2(x^2 - 4x + 4 - 4) + 3 = -2(x - 2)^2 + 11: \text{max } 11 \text{ at } x = 2.$$

**TRAP**  $-2(x^2 - 4x + 4 - 4)$ : the -4 comes out as  $(-2)(-4) = +8$ , not -4.

### 3 Optimization

**USE** Write the quantity to optimize as a quadratic, find the vertex, then answer the question that was asked.

**EXAMPLE**  $R = (20 + 2x)(100 - 5x)$  peaks at  $x = 5$ : price  $20 + 2(5) = \$30$ , revenue \$2250.

**TRAP**  $x = 5$  is the number of increases, not the price: substitute it back.

### 4 Radicals

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (a - \sqrt{b})(a + \sqrt{b}) = a^2 - b$$

**USE** Simplify with the **largest** perfect-square factor. Add like radicals only. Rationalize with the conjugate.

**EXAMPLE**  $\sqrt{48} = 4\sqrt{3}$ ;  
 $\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$ .

#### TRAP

$$(3 + \sqrt{2})^2 = 9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}, \text{ not } 11.$$

**TRAP**  $(5 - \sqrt{2})(5 + \sqrt{2}) = 25 - 2 = 23$ , not 27.

### 5 Inverse of a quadratic

$$f^{-1}(x) = h \pm \sqrt{\frac{x - k}{a}}$$

**USE** Restrict  $f$  to  $x \geq h$  (keep +) or  $x \leq h$  (keep -) so  $f^{-1}$  is a function. The graph reflects over  $y = x$ .

**EXAMPLE**  $f(x) = 2(x - 1)^2 + 3, x \geq 1$ :  
 $f^{-1}(x) = 1 + \sqrt{(x - 3)/2}$ .

**TRAP** Restrict the **domain** with  $h$ : vertex  $(5, -3)$  needs  $x \geq 5$ , not  $x \geq -3$ .

### 6 Solving

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**USE** Get 0 on one side first. Factor if you can; otherwise complete the square or use the formula.

**TRAP**  $(x - 2)(x + 1) = 4$  does not give  $x - 2 = 4$ : expand to  $x^2 - x - 6 = 0$ , so  $x = 3$  or  $-2$ .

**TRAP** With  $b = -5, b^2 = (-5)^2 = 25$ , never -25.

### 7 The discriminant

$$D = b^2 - 4ac:$$

$$D > 0$$

**two real roots**

$$D = 0$$

**one (double) root**

$$D < 0$$

**no real roots**

**TIP** In context, reject roots that make no sense, like a negative time or length.

### 8 Families

**USE** Same vertex:  $y = a(x - h)^2 + k$ . Same zeros:  $y = a(x - r)(x - s)$ . Substitute one more point to find  $a$ .

**EXAMPLE** Zeros -3 and 2, through  $(0, 12)$ :  $12 = a(3)(-2)$ , so  
 $y = -2(x + 3)(x - 2)$ .

**TRAP** A zero of -3 gives the factor  $(x + 3)$ , not  $(x - 3)$ .

### 9 Line and parabola

**USE** Set the equations equal and collect **every** term on one side. The discriminant of that equation counts the intersections: 2 secant, 1 tangent, 0 none.

**EXAMPLE**  $x^2 + x - 1 = 3x + 2$ :  
 $x^2 - 2x - 3 = 0$ , so  $(3, 11)$  and  $(-1, -1)$ .

**TRAP** Use the discriminant of the combined equation, not of the parabola alone.