

Quadratic Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 50 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Three forms: vertex $y = a(x - h)^2 + k$ shows the vertex (h, k) ; factored $y = a(x - r)(x - s)$ shows the zeros r, s and the axis $x = \frac{r+s}{2}$; standard $y = ax^2 + bx + c$ shows the y -intercept c .

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The discriminant $b^2 - 4ac$ is positive (two real roots), zero (one) or negative (none).

Radicals: $\sqrt{ab} = \sqrt{a}\sqrt{b}$, but $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ and $(a + \sqrt{b})^2 \neq a^2 + b$.

Show your work on separate paper.

Core Skills (K)

Three Forms · Max & Min · Radicals · Solving · Families & Systems

K1 Three Forms

- For $y = -2(x + 3)^2 + 5$, state the vertex, the axis of symmetry and the direction of opening.
- Determine the y -intercept of $y = -2(x + 3)^2 + 5$.
- For $y = 3(x - 1)(x + 5)$, state the zeros, the axis of symmetry and the vertex.

K2 Completing the Square

Write each function in vertex form. Then state its maximum or minimum value and the value of x where it occurs.

- $y = x^2 - 10x + 18$
- $y = 2x^2 + 12x + 7$
- $y = -3x^2 + 12x - 5$

K3 Radicals

Simplify fully. Leave no radical in a denominator.

- $\sqrt{48} + \sqrt{27}$
- $(3 - \sqrt{2})^2$
- $\frac{4}{3 - \sqrt{5}}$

K4 Three Ways to Solve

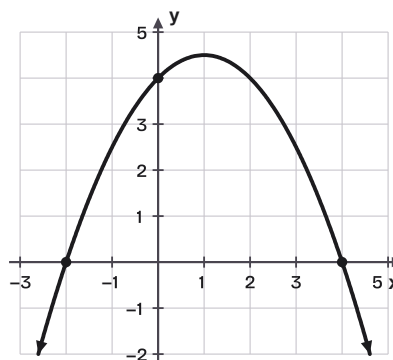
- Solve $x^2 - 2x = 15$ by factoring.
- Solve $x^2 + 6x - 4 = 0$ by completing the square. Give exact answers.
- Solve $2x^2 - 4x - 3 = 0$ with the quadratic formula. Give exact answers in simplest form.

K5 Families & Systems

Completing the square: factor a from the x^2 and x terms only. The number subtracted inside the bracket is multiplied by a when it comes out.

Line and parabola: set the two equations equal and collect every term on one side. The discriminant of *that* equation counts the intersection points.

- Determine the equation of the parabola graphed below, in factored form. Each grid square is 1 unit.



- Find the member of the family $y = a(x - 1)^2 + 3$ that passes through $(3, -5)$.
- Solve the system $y = x^2 - 3x + 1$ and $y = x - 2$.

Real World (A)

Optimization · Inverses in Context · Line Meets Parabola

A1 Muffin Pricing

A café in Kingston tracks its daily muffin sales. Each \$0.25 increase in price means 8 fewer muffins sold.

Price (dollars)	2.50	2.75	3.00	3.25
Muffins sold	160	152	144	136

- Let x be the number of \$0.25 increases. Write the daily revenue $R(x)$, then expand it to standard form.
- Determine the price that maximizes daily revenue, and the maximum revenue.
- The owner wants \$500 a day from muffins. How many \$0.25 increases will do it?

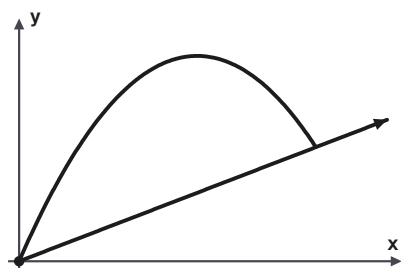
A2 Skid Marks

On dry pavement, a car travelling at v km/h needs about $d(v) = 0.005v^2$ metres to brake to a stop, for $v \geq 0$.

- Determine $d^{-1}(x)$. What do its input and its output represent?
- After a collision in a 70 km/h zone, police measure skid marks 32 m long. Use d^{-1} to decide whether the driver was speeding.
- The equation $0.005v^2 = 32$ has two solutions. State both, and say which one the police can use and why.

A3 Uphill Golf

A golf ball is hit from the bottom of a hill, at the origin. Its path is $y = -0.02x^2 + 1.2x$ and the hillside is the line $y = 0.2x$, where x is the horizontal distance and y the height, both in metres. The sketch is not to scale.



- Determine the point where the ball lands on the hillside.
- Determine the greatest vertical distance between the ball and the hillside, and the value of x where it occurs.
- A tree 10 m tall grows on the hillside at $x = 40$. Does the ball clear it?

Think It Through (T)

Error Analysis · Counterexamples · Tangent Lines

T1 Spot the Flaw

A student finds the inverse of $f(x) = 2(x - 3)^2 - 8$, restricted to $x \leq 3$:

"Swap x and y : $x = 2(y - 3)^2 - 8$, so $\frac{x+8}{2} = (y - 3)^2$.

Therefore $f^{-1}(x) = 3 + \sqrt{\frac{x+8}{2}}$, with domain $x \leq 3$ and range $y \geq -8$."

- Evaluate $f(1)$, then substitute the result into the student's formula. What does this show?
- Identify the two errors. Then write the correct $f^{-1}(x)$ with its domain and range.
- Evaluate $f^{-1}(10)$ and verify that $f(f^{-1}(10)) = 10$.

T2 Line Meets Parabola

Let $p(x) = x^2 - 4x + 5$.

- A student claims: "The discriminant of p is negative, so no line can meet the graph of p twice." Give a counterexample and its two meeting points.
- Determine k so that the line $y = 2x + k$ is tangent to the graph of p , and state the point of tangency.
- For which values of k does the line $y = 2x + k$ miss the graph of p entirely?

Boss Level

Optional challenge

Vertices on a Line

For each real number k , the parabola P_k has equation $y = x^2 - 2kx + k^2 + k - 6$.

- Write P_k in vertex form. Find the equation of the line that contains the vertex of every P_k .
- Determine all values of k for which P_k has two x -intercepts.
- Determine the value of k for which the two x -intercepts of P_k are 4 units apart.



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