

Polynomials and Rational Expressions

Ontario MCR3U • polynomial operations, factoring, simplifying and operating on rational expressions

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Worksheet, answers &
this cheat sheet



MOST TESTED Simplify $\frac{x^2 - 9}{x^2 + x - 6}$ and state the restrictions. (card 4)

Which factoring?

Every term
shares a factor

Common factor

Two squares,
subtracted

$(a - b)(a + b)$

Three terms

Product and sum

1 Adding and multiplying

USE Collect like terms: same variables, same exponents. Multiply each term of one bracket by each term of the other.

EXAMPLE

$$(3x - 2)(x + 5) = 3x^2 + 15x - 2x - 10 = 3x^2 + 13x - 10$$

TRAP A minus before a bracket flips every sign:

$$(4x^2 - x) - (x^2 - 3x + 2) = 3x^2 + 2x - 2.$$

2 Special products

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

USE Three factors: multiply two of them, then multiply the result by the third.

TRAP $(x - 2)^2 = x^2 - 4x + 4$, not $x^2 - 4$: squaring a binomial keeps the middle term.

3 Factoring

USE Common factor first. $ax^2 + bx + c$: two numbers with product ac and sum b , then group. Four terms: group in pairs. Check by expanding.

EXAMPLE $2x^2 + 5x - 3$: 6 and -1 , so
 $2x^2 + 6x - x - 3 = (x + 3)(2x - 1)$.

TRAP $4x^2 - 49 = (2x - 7)(2x + 7)$, not $(4x - 7)(4x + 7)$: take the root of the coefficient too.

TRAP $-6x^2 + 9x = -3x(2x - 3)$: divide every term by the negative factor.

4 Simplifying

USE Factor fully. State restrictions from the original denominator. Then cancel common factors.

EXAMPLE

$$(x^2 - 9)/(x^2 + x - 6) = (x - 3)(x + 3)/[(x + 3)(x - 2)] = (x - 3)/(x - 2),$$

$$x \neq -3, 2.$$

TRAP Cancel factors, never terms: $(x + 5)/(x + 7)$ does not simplify to $5/7$.

5 Holes and asymptotes

USE A factor that cancels gives a **hole**. A factor left in the denominator gives a vertical asymptote.

EXAMPLE $y = (x - 3)/(x - 2)$ from card 4: Hole at $(-3, 6/5)$, asymptote $x = 2$.

TRAP The simplified graph still has the hole: draw an open circle at $x = -3$.

6 Multiplying and dividing

USE Factor, flip each divisor, then cancel. Restrictions: every original denominator and the numerator of each divisor.

TRAP $(2 - x)/(x - 2) = -1$, not 1:
 $2 - x = -(x - 2)$. Keep $x \neq 2$.

TRAP Dividing by $(x - 4)/(x + 1)$ adds $x \neq 4$ as well as $x \neq -1$.

7 Adding and subtracting

USE Factor the denominators. LCD: each factor once, at its highest power. Rewrite, combine the numerators in brackets, simplify. Check with an allowed value.

EXAMPLE

$$2/x + 3/(x + 1) = (5x + 2)/[x(x + 1)],$$

$$x \neq 0, -1.$$

TRAP Never add denominators:

$$1/x + 1/y = (x + y)/(xy), \text{ not } 2/(x + y).$$

TRAP Check with a value that isn't restricted: $x = 0$ proves nothing here.