

Polynomials and Rational Expressions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 50 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Like terms have the same variables with the same exponents. A minus sign in front of a bracket changes the sign of **every** term inside it.

Restrictions come from every denominator of the original expression, and from the numerator of any divisor. State them **before** you cancel.

Cancel **factors**, never terms: $\frac{x+5}{x+7}$ does not simplify to $\frac{5}{7}$.

Show your work on separate paper.

Core Skills (K)

Polynomials · Factoring · Simplifying · Multiplying & Dividing · Adding & Subtracting

K1 Expand and Simplify

Expand and simplify.

- $(3x^2 - 5x + 2) - (x^2 - 4x - 6)$
- $5x - 2x(x - 4)$
- $(2x - 3)^2 - (x + 2)(x - 2)$

K2 Factor Fully

Factor fully.

- $-4x^3 + 12x^2 - 8x$, starting with a negative common factor
- $6x^2 + x - 12$
- $x^3 - 3x^2 - 4x + 12$

K3 Simplify and Restrict

Simplify each expression and state its restrictions.

- $\frac{x^2 - 9}{x^2 + x - 6}$
- $\frac{10 - 2x}{x^2 - 25}$
- $\frac{2x^2 + 5x - 3}{4x^2 - 1}$

K4 Multiply and Divide

Simplify each expression and state its restrictions.

- $\frac{x^2 - 4}{3x} \cdot \frac{6x^2}{x^2 + x - 2}$
- $\frac{x + 3}{x - 1} \div \frac{x^2 - 9}{x^2 - 1}$
- $\frac{2x}{x + 4} \div \frac{x^2}{x^2 - 16} \cdot \frac{x + 1}{x - 4}$

Special products: $(a \pm b)^2 = a^2 \pm 2ab + b^2$ and

$(a + b)(a - b) = a^2 - b^2$. Factor fully: common factor first, then trinomials, difference of squares or grouping.

Add or subtract over the lowest common denominator; never add the denominators.

K5 Add and Subtract

Simplify each expression and state its restrictions.

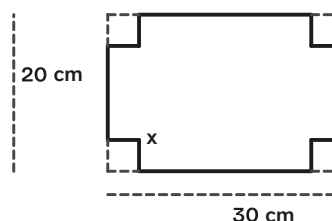
- $\frac{3x + 1}{x - 2} - \frac{x - 5}{x - 2}$
- $\frac{4}{x} + \frac{3}{x + 2}$
- $\frac{3x + 4}{x^2 + x - 6} - \frac{2}{x - 2}$

Real World (A)

Polynomial Models · Rational Models

A1 Open-Top Box

An open-top box is made from a 30 cm by 20 cm sheet of card: a square of side x cm is cut from each corner, then the sides are folded up. Let $V(x)$ be the volume of the box in cubic centimetres.



- Write $V(x)$ as a product of three factors, expand it into standard form, and find the volume when $x = 5$.
- Factor your standard form fully. Which values of x make $V(x) = 0$?
- Which of these zeros, if any, describe a box you could actually fold? State the realistic domain of V .

A2 Paddling the Grand River

A canoeist paddles 6 km upstream on the Grand River, then 6 km back downstream. The current flows at 2 km/h, and the canoeist paddles at s km/h in still water.

- Write the total trip time $T(s)$, in hours, as a single rational expression.
- At $s = 4$, find the total time and how much longer the upstream leg takes than the downstream leg.
- The expression gives $T(1) = -4$. What does this mean for the canoeist? State the realistic domain of T .

A3 Token Draw

At a school fair, a bag holds x green tokens and 2 orange tokens. Two tokens are drawn, one after the other, without putting the first one back. The first is green with probability $\frac{x}{x+2}$. If it is green, the second is green with probability $\frac{x-1}{x+1}$ and orange with probability $\frac{2}{x+1}$.

- Multiply to write $P(\text{green, green})$ and $P(\text{green, orange})$ as single rational expressions.
- Simplify $P(\text{green, green}) \div P(\text{green, orange})$. With 9 green tokens, how many times as likely is green-green as green-orange?
- The simplified ratio gives $-\frac{1}{2}$ when $x = 0$. What does this tell you about a bag with no green tokens?

Think It Through (T)

Error Analysis · Building Expressions

T1 The Check That Checks Nothing

A student simplifies an expression:

$$\frac{x}{x-3} - \frac{x+15}{x^2-9} = \frac{x(x+3) - x+15}{(x-3)(x+3)} = \frac{x^2+2x+15}{(x-3)(x+3)}$$

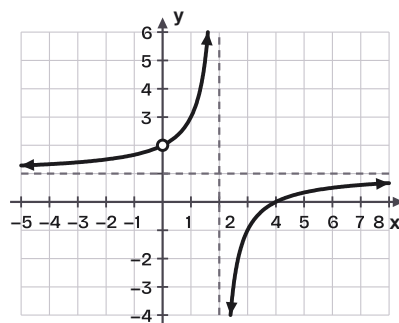
They test both versions in a table and write: "The rows match, so my answer is correct."

x	3	-3
Original	undefined	undefined
Student's answer	undefined	undefined

- Find the error in the student's work. Then simplify the expression correctly and state its restrictions.
- Explain why the student's table proves nothing.
- Choose an allowed value of x and use it to show that the student's answer is wrong and yours is right.

T2 Reverse-Engineer It

The graph of a rational function f is shown, with its asymptotes dashed and an open circle at $(0, 2)$. Each grid square is 1 unit.



- Write an equation for f as a quotient of factored polynomials.
- Write a rational expression that simplifies to $\frac{2}{x+5}$ and whose restrictions are exactly $x \neq \pm 5$.
- Write two different rational expressions that both simplify to $\frac{x+1}{x-3}$ but have different restrictions. State the restrictions of each.

Boss Level

Optional challenge

A Sum That Factors

A sum of two squares such as $x^4 + 4$ can still factor, with one extra idea.

- Show that $x^4 + 4 = (x^2 + 2)^2 - (2x)^2$. Then factor $x^4 + 4$ into two quadratics.
- Find every positive integer n for which $n^4 + 4$ is prime.
- Without a calculator, evaluate $\frac{(3^4 + 4)(7^4 + 4)}{(1^4 + 4)(5^4 + 4)}$.



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