

Sequences and Series

Ontario MCR3U • arithmetic and geometric sequences and series, recursion, Pascal's triangle

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Worksheet, answers &
this cheat sheet



MOST TESTED Find the sum of the first 10 terms of $3 + 6 + 12 + \dots$. (card 6)

Which sequence?

Constant difference

Arithmetic

Constant ratio

Geometric

Each term from the
two before

Recursive

1 Arithmetic sequences

$$t_n = a + (n - 1)d \quad d = t_n - t_{n-1}$$

USE Check d for **every** consecutive pair.

Finding n : solve for it; it must be a positive integer.

TRAP $a = 20, d = -3$:

$$t_{15} = 20 + 14(-3) = -22, \text{ not } 20 + 14 - 3.$$

TRAP \$45 000 now, +\$1500 a year:
after 10 years is 45 000 + 10(1500), the
11th term.

2 Geometric sequences

$$t_n = ar^{n-1} \quad r = \frac{t_n}{t_{n-1}}$$

USE Check r for every pair. The exponent
applies to r only. Growth of 4%: $r = 1.04$.

TRAP $a = 5, r = 2$: $t_4 = 5(2)^3 = 40$, not
 $(5 \cdot 2)^3 = 1000$.

TRAP $486 = 6(3)^{n-1}$: $3^{n-1} = 81 = 3^4$, so
 $n - 1 = 4$ and $n = 5$.

3 Recursive formulas

$$t_1 = a, \quad t_n = t_{n-1} + d \quad t_1 = a, \quad t_n = r t_{n-1}$$

USE To go explicit, keep the same a and d
(or r) and keep $n - 1$.

EXAMPLE $t_1 = 15, t_n = t_{n-1} - 6$:

$$t_n = 15 - 6(n - 1) = 21 - 6n.$$

TRAP $t_1 = 1, t_2 = 3, t_n = t_{n-1} + t_{n-2}$
gives 1, 3, 4, 7, 11: start from the given
terms, not Fibonacci's.

4 Classifying

USE Test differences **and** ratios. Constant
difference: arithmetic. Constant ratio:
geometric. Neither: look for another rule.

TRAP 4, 8, 16, 32: the differences
change, but the ratio is 2, so it is
geometric, not "neither".

5 Arithmetic series

$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}[2a + (n - 1)d]$$

USE Find n first. Solving for n can give a
quadratic: keep only positive integers. Use
one unit (dollars or cents).

EXAMPLE $5 + 8 + \dots + 62$:

$$62 = 5 + 3(n - 1), \quad n = 20,$$

$$S_{20} = 10(67) = 670.$$

TRAP In $2a + (n - 1)d$, multiply $(n - 1)d$ first, then add $2a$.

6 Geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

USE Bouncing ball: the first drop, then each
bounce counts **up and down**.

EXAMPLE $3 + 6 + 12 + \dots$:

$$S_{10} = 3(2^{10} - 1)/(2 - 1) = 3069.$$

TRAP $3^n = 243$ gives $n = 5$, not
 $243/3 = 81$: find the power.

TRAP A ball dropped 8 m, rebounding to
 $3/4$: by the 4th landing it travels $8 +$
 $2(6 + 4.5 + 3.375) = 35.75$ m.

7 Pascal's triangle

USE Row n ($1, n, \dots, n, 1$) gives the
coefficients of $(a + b)^n$. Powers of a fall as
powers of b rise; each term's exponents add
to n .

EXAMPLE

$$(x + 2)^3 = x^3 + 3x^2(2) + 3x(2)^2 + 2^3 = x^3 + 6x^2 + 12x + 8.$$

TRAP In $(2x - 3)^3$, $(2x)^3 = 8x^3$, not $2x^3$:
the power applies to the coefficient too.