

Sequences and Series Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 50 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Test every consecutive pair: a constant difference $d = t_n - t_{n-1}$

means arithmetic; a constant ratio $r = \frac{t_n}{t_{n-1}}$ means geometric.

Geometric: $t_n = ar^{n-1}$ and $S_n = \frac{a(r^n - 1)}{r - 1}$, $r \neq 1$.

In ar^{n-1} the exponent applies to r only: ar^{n-1} is **not** $(ar)^{n-1}$. In the same way, $(3x)^2 = 9x^2$, not $3x^2$.

Arithmetic: $t_n = a + (n - 1)d$ and

$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}[2a + (n - 1)d].$$

Pascal's triangle: row n (the row 1, $n, \dots, n, 1$) gives the coefficients of $(a + b)^n$. Each entry is the sum of the two entries above it.

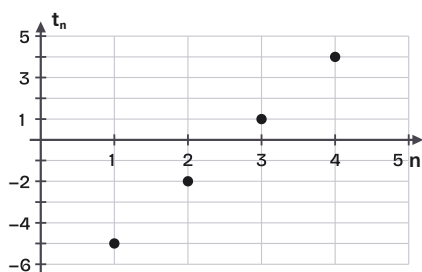
Show your work on separate paper.

Core Skills (K)

Arithmetic · Geometric · Recursive · Arithmetic Series · Geometric Series

K1 Arithmetic Sequences

- a) The first four terms of an arithmetic sequence are graphed below. Each grid square is 1 unit. Determine d and t_{20} .



- b) Determine the general term of 41, 34, 27, ... and find t_{15} .
- c) Which term of 150, 143, 136, ... is equal to 3?

K2 Geometric Sequences

- a) Determine r and t_{10} for 3, -6, 12, ...
- b) A geometric sequence has $a = 5$ and $r = 3$. Determine its general term and evaluate t_6 .
- c) Which term of 4, 12, 36, ... is equal to 2916?

K3 Recursive Sequences

- a) Write t_3 to t_6 for $t_1 = 2$, $t_2 = 5$, $t_n = t_{n-1} + t_{n-2}$.
- b) Determine the explicit formula for $t_1 = 9$, $t_n = t_{n-1} + 4$.

- c) For the sequence in the table, state whether it is arithmetic, geometric or neither. Then write a recursive formula for it.

n	1	2	3	4	5
t_n	3	7	15	31	63

K4 Arithmetic Series

- a) Find the sum $7 + 11 + 15 + \dots + 87$.
- b) Find S_{25} for the arithmetic series with $a = -6$ and $d = 3$.
- c) An arithmetic series has $a = 3$ and $d = 4$. How many terms add to 300?

K5 Geometric Series

- a) Find S_8 for $2 + 6 + 18 + \dots$
- b) How many terms of $5 + 10 + 20 + \dots$ add to 1275?
- c) Find S_6 for $64 - 32 + 16 - \dots$

Real World (A)

Savings · Chain Messages · Bouncing Ball

A1 Laptop Fund

Noah saves \$5.00 in the first week of the year. Each week after that, he saves \$0.50 more than the week before.

- a) How much does Noah save in week 20?
- b) How much has he saved in total after 20 weeks?
- c) Noah wants \$1000 for a laptop by the end of the year (52 weeks). Solve $S_n = 1000$. Can he reach his goal in time? How much does he have after 52 weeks?

A2 Chain Message

In round 1 of a chain message, 4 people receive it. In each later round, every person who received it in the previous round sends it to 3 people who have not seen it yet.

- Write the general term for the number of new people who receive the message in round n . How many receive it in round 6?
- In which round do more than 100 000 new people first receive the message?
- Ontario has about 16 million people. In which round does the formula first give more new people than that? What does this mean for the model?

A3 Bouncing Ball

A ball is dropped from a height of 10 m. After each bounce, it rises to 60% of the height it fell from.

- Find the height of the 4th rebound.
- Find the total vertical distance the ball has travelled when it hits the ground for the 6th time. Round to the nearest hundredth of a metre.
- The formula gives a positive rebound height for every bounce, so it predicts the ball never stops. Find the first rebound lower than 1 cm, and say what this means for the model.

Think It Through (T)

Error Analysis · Building Sequences

T1 Spot the Flaw

A student expands $(3x - 2)^4$:

"Row 4 of Pascal's triangle is 1, 4, 6, 4, 1, so

$$(3x - 2)^4 = (3x)^4 + 4(3x)^3(-2) + 6(3x)^2(-2)^2 + 4(3x)(-2)^3 + (-2)^4,$$

which is $3x^4 - 24x^3 + 72x^2 - 96x + 16$."

- Evaluate $(3x - 2)^4$ and the student's answer at $x = 1$. What does the result prove?
- Identify the error and write the correct expansion.
- Explain why the student's x -term and constant term are correct anyway.

T2 Build It Backwards

Build each sequence from the information given. Some parts have more than one answer.

- An arithmetic sequence has $t_5 = 23$ and $t_{12} = 51$. Determine its general term.
- A geometric sequence has $t_2 = 6$ and $t_4 = 54$. Determine every possible general term.
- Find every value of x for which 2, x , 18 is a geometric sequence, and every value for which it is an arithmetic sequence.

Boss Level

Optional challenge

Neither 3 nor 5

Use arithmetic series to answer each part.

- Find the sum of all multiples of 3 from 3 to 300.
- Find the sum of all multiples of 5 from 5 to 300.
- Find the sum of all integers from 1 to 300 that are divisible by neither 3 nor 5.



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