

Sinusoidal Functions

Ontario MCR3U • periodic functions, sine and cosine graphs, transformations, modelling

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Worksheet, answers &
this cheat sheet



MOST TESTED A sinusoid has a maximum of 7 at $x = 30^\circ$, a minimum of -1 and a period of 180° . Write its equation. (card 5)

Sine or cosine?

Starts on the
axis, going up

$$y = a \sin(kx) + c$$

Starts at a
maximum

$$y = a \cos(kx) + c$$

Starts at a
minimum

$$y = -a \cos(kx) + c$$

1 Periodic functions

$$\text{amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{axis } y = \frac{\text{max} + \text{min}}{2}$$

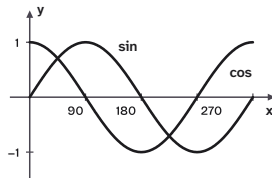
USE Period: the horizontal length of one full cycle, back to the same height and heading the same way.

TRAP Crossing the axis again is not a full cycle unless the curve is moving the same way as at the start.

TRAP A tide between 2 m and 10 m has amplitude 4 m (not 8) and axis $y = 6$ (not 5).

2 $\sin x$ and $\cos x$

USE Unit circle point: $(\cos \theta, \sin \theta)$. Both: amplitude 1, period 360° , range $[-1, 1]$. Sine starts on the axis going up; cosine starts at a maximum. $\cos x = \sin(x + 90^\circ)$.



TRAP $\cos 120^\circ = -\cos 60^\circ = -1/2$: the related angle is 60° , and cosine is negative in QII.

3 Transformations

$$y = a \sin[k(x - d)] + c$$

$$\text{period} = \frac{360^\circ}{|k|}$$

USE Amplitude $|a|$, phase shift d , axis $y = c$, range $[c - |a|, c + |a|]$. Factor k out first.

TRAP $\sin(2x - 60^\circ) = \sin[2(x - 30^\circ)]$: the shift is 30° right, not 60° .

TRAP $k = 1/3$: period $360^\circ \div (1/3) = 1080^\circ$, not 120° .

4 Graphing with key points

USE Five key points per cycle, a quarter period apart. Sine: axis, max, axis, min, axis. Map each with $(x, y) \rightarrow (x/k + d, ay + c)$.

EXAMPLE $y = 2 \sin[3(x - 10^\circ)] + 1$:
 $(0, 0) \rightarrow (10^\circ, 1)$ and $(90^\circ, 1) \rightarrow (40^\circ, 3)$.

5 Equation from a graph

USE $a = (\text{max} - \text{min})/2$,
 $c = (\text{max} + \text{min})/2$, $k = 360^\circ/\text{period}$. d : a maximum for cosine, a rising axis crossing for sine. Keep the brackets.

EXAMPLE Max 7 at 30° , min -1 , period 180° : $y = 4 \cos[2(x - 30^\circ)] + 3$.

TRAP A period of 120° gives
 $k = 360^\circ/120^\circ = 3$, not $k = 120$.

6 Modelling

USE Ferris wheel: amplitude = radius, axis = hub height. Boarding at the bottom:
 $h = -a \cos(kt) + c$. Use time units that match the period.

EXAMPLE Radius 12 m, hub 14 m, one turn in 60 s: $h = -12 \cos(6t) + 14$.

TRAP In $h = -12 \cos(6t) + 14$, 14 m is the hub height; boarding is at $14 - 12 = 2$ m.

7 Solving with a model

USE Isolate the trig function, find **both** angles in the cycle, then solve for t . Add multiples of the period for later cycles.

EXAMPLE $\cos(6t) = 0.5$: $6t = 60^\circ$ or 300° , so $t = 10$ s or 50 s in the first turn.

TRAP The calculator shows one angle; the second angle in the cycle is an answer too.