

Sinusoidal Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 55 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Amplitude = $\frac{\max - \min}{2}$, always positive. **Equation of the axis:**

$y = \frac{\max + \min}{2}$, written as an equation.

Factor k out first: $\sin(2x - 60^\circ) = \sin(2(x - 30^\circ))$ is a shift of 30° right, not 60° .

Solving $\sin \theta = \text{value}$ gives **two** angles per cycle. Find both, then add multiples of the period.

Show your work on separate paper.

Core Skills (K)

Periodic Functions · Sine & Cosine · Transformations · Equations from Graphs

K1 Read the Cycle

The graph shows a periodic function f . Each grid square is 1 unit.



- State the period.
- State the amplitude and the equation of the axis.
- Determine $f(44)$.

K2 Sine & Cosine Values

Do not use a calculator.

- Evaluate $\sin 240^\circ$ and $\cos 240^\circ$ exactly.
- Evaluate $\cos 135^\circ$ and $\sin 330^\circ$ exactly.
- For $y = \cos x$, $0^\circ \leq x \leq 360^\circ$, state the zeros, the maximum points and the minimum point.

K3 Read the Equation

State the requested features of each function.

- $y = -3 \sin(2(x - 30^\circ)) + 1$: amplitude, period, phase shift and equation of the axis.
- $y = 4 \cos\left(\frac{1}{3}x\right) - 2$: amplitude, period and range.
- $y = \sin(3x + 90^\circ)$: period and phase shift.

For $y = a \sin(k(x - d)) + c$: amplitude $|a|$, period $\frac{360^\circ}{|k|}$, phase shift d (right when $d > 0$), axis $y = c$.

$\cos x = \sin(x + 90^\circ)$. A sine curve starts a cycle on its axis going up; a cosine curve starts at a maximum.

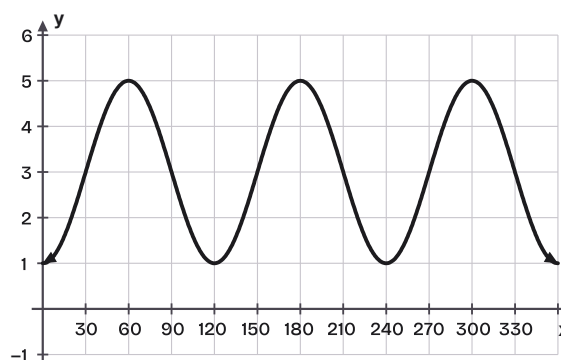
K4 Key Points

Let $y = -2 \sin(2(x + 45^\circ)) + 3$.

- Determine the five key points of one cycle, starting at $x = -45^\circ$.
- Evaluate y at $x = 15^\circ$ exactly.
- Find all x with $0^\circ \leq x \leq 360^\circ$ where y reaches its maximum value.

K5 Equation from a Graph

The graph shows a sinusoidal function, with x in degrees. Vertical grid lines are 30° apart and horizontal grid lines are 1 unit apart.



- State the maximum, the minimum, the amplitude and the equation of the axis.
- Determine the period and the value of k .
- Determine an equation of the form $y = a \sin(k(x - d)) + c$.

Real World (A)

Periodic Models · Solving with Models

A1 Heartbeat

At rest, Jordan's blood pressure rises and falls with each heartbeat, between 80 mmHg and 120 mmHg. Their heart beats 75 times per minute. Model the pressure P , in mmHg, as a sinusoidal function of the time t , in seconds.

- State the amplitude, the equation of the axis and the period, with units.
- The pressure is at its maximum at $t = 0$. Determine $P(t)$ as a cosine function, then evaluate $P(0.3)$.
- During a workout, Jordan's heart rate climbs steadily from 75 to 150 beats per minute. Which parameter of your model changes, and what is its value at 150 beats per minute? Can one function of this form model the whole workout?

A2 Ferris Wheel

A Ferris wheel at a summer fair in Toronto has a radius of 12 m, and its axle is 14 m above the ground. It turns once every 80 s. A rider boards at the lowest point at $t = 0$.

- Determine $h(t)$, the rider's height in metres after t seconds.
- Find the height at $t = 30$ s, exactly and to one decimal place.
- Find the first two times the rider is 20 m above the ground.
- A poster says riders reach 28 m at the top. Use the model to find when the rider is 28 m high, and interpret the result.

A3 Bay of Fundy Tides

The water depth d at a dock on the Bay of Fundy, Nova Scotia, was recorded every 3 hours starting at midnight ($t = 0$).

t (h)	0	3	6	9	12	15
d (m)	8	14	8	2	8	14

- State the amplitude, the period and the equation of the axis, with units.
- Determine an equation for $d(t)$ using a cosine function.
- Find all times with $0 \leq t \leq 12$ when the depth is exactly 5 m.
- A cargo boat needs at least 5 m of water for its whole 2-hour unloading at the dock. During the first 12 hours, at what times can it start unloading?

Think It Through (T)

Error Analysis · Building Functions

T1 Spot the Flaw

A student converts $y = 5 \cos(2(x - 10^\circ)) + 8$ into a sine function:

"Cosine is sine shifted by 90° , so I subtract 90° from the phase shift: $y = 5 \sin(2(x - 100^\circ)) + 8$."

- Evaluate both functions at $x = 10^\circ$. Are they the same function?
- Identify the error and write a correct sine equation.
- A second student writes $y = -5 \sin(2(x - 55^\circ)) + 8$. Show that this is also correct.

T2 Build It Backwards

A sinusoidal function has a maximum value of 7, a minimum value of -1 , a period of 90° , and a maximum at $x = 30^\circ$.

- Write its equation in the form $y = a \sin(k(x - d)) + c$ with $a > 0$.
- Write the same function in the form $y = a \cos(k(x - d)) + c$ with $a > 0$.
- A student says the value of d in (a) is the only one possible. Explain why other values work, and give two of them.

Boss Level

Optional challenge

Counting Solutions

Let $f(x) = 4 \sin(3x) + 2$ for $0^\circ \leq x \leq 360^\circ$.

- Solve $f(x) = 4$.
- Without solving, find the sum of all solutions of $f(x) = k$ for any k with $2 < k < 6$.
- Find the value of k for which $f(x) = k$ has exactly 7 solutions.



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