

Transformations of Functions

Ontario MCR3U • translations, reflections, stretches, combined transformations

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Worksheet, answers &
this cheat sheet



MOST TESTED Find the image of $(4, 5)$ under $y = -2f(3x + 6) - 1$, and describe the transformations. (card 4)

Inside or outside?

a and c , outside
the brackets

y , as written

k and d , inside the
brackets

x , reversed

A negative sign

Reflection

1 Translations

$$y = f(x - d) + c \quad (x, y) \rightarrow (x + d, y + c)$$

USE $d > 0$: right d . $c > 0$: up c . Every point moves, not only the intercepts.

EXAMPLE $y = (x + 3)^2 - 4$: left 3, down 4, so the vertex is $(-3, -4)$.

TRAP $f(x + 3)$ moves **left** 3: the sign inside the brackets works the opposite way.

2 Reflections

$$-f(x): \text{in the } x\text{-axis} \quad f(-x): \text{in the } y\text{-axis}$$

USE Invariant points lie on the mirror axis.

$$-f(-x) \text{ does both: } (x, y) \rightarrow (-x, -y).$$

Even: $f(-x) = f(x)$. Odd: $f(-x) = -f(x)$.

TRAP Test **every** term: $2x^3 - 5x + 3$ is neither even nor odd, because the $+3$ never changes sign.

TRAP $-f(x)$ negates the whole rule:
 $-(2\sqrt{x-1} - 4) = -2\sqrt{x-1} + 4$.

3 Stretches

$$af(x): y \times a \quad f(kx): x \div k$$

USE $|a| > 1$ stretches, $0 < |a| < 1$ compresses. $f(kx)$ scales horizontally by $1/k$. Invariant points: x -intercepts under $af(x)$; the y -intercept under $f(kx)$.

TRAP $f(2x)$ is a horizontal **compression** by $1/2$: divide the x -coordinates by 2.

TRAP $3f(x)$ multiplies every y : for $f(x) = x^2 - 2$, $3f(x) = 3x^2 - 6$, not $3x^2 - 2$.

4 Combined: the mapping rule

$$y = af[k(x - d)] + c$$

$$(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

USE Factor k out first. Stretches and reflections, then translations.

EXAMPLE

$$y = 3f(2x - 6) + 4 = 3f[2(x - 3)] + 4:$$

$$(4, 1) \rightarrow (4/2 + 3, 3 + 4) = (5, 7).$$

TRAP $f(2x - 6) = f[2(x - 3)]$ shifts right 3, not 6: factor before reading d .

5 Domain and range

USE Domain: map the x -endpoints with $x/k + d$. Range: map the y -endpoints with $ay + c$. A negative a or k swaps the ends.

EXAMPLE Domain $[-2, 6]$,
 $y = f(2x - 4) = f[2(x - 2)]$: new
domain $[1, 5]$.

TRAP Range $y \geq 0$ under $y = -3f(x) + 1$ becomes $y \leq 1$: the reflection flips the inequality.

6 Reading an equation

USE Factor, then read a (vertical; x -axis reflection if negative), k (horizontal by $1/|k|$; y -axis reflection if negative), d , c .

EXAMPLE

$$3f(-2x + 6) - 4 = 3f[-2(x - 3)] - 4:$$

stretch 3, compress $1/2$, reflect in the y -axis, right 3, down 4.

TRAP Compress by $1/3$, then right 2:
 $f[3(x - 2)]$, not $f(3x - 2)$, which moves right only $2/3$.

7 Equation from a graph

USE Name the parent. Read (d, c) from the vertex or endpoint. Substitute a **second** point to find a .

EXAMPLE Vertex $(2, -3)$, through $(4, 5)$:
 $5 = a(4 - 2)^2 - 3$, so $a = 2$:
 $y = 2(x - 2)^2 - 3$.

TRAP Substituting the vertex gives $-3 = -3$ and no a : use another point.