

Transformations of Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 50 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Outside vs inside: a and c act on y as written. k and d act on x the opposite way: $f(x - 3)$ moves right 3, $f(2x)$ compresses horizontally by a factor of $\frac{1}{2}$.

Reflections: $y = -f(x)$ reflects in the x -axis, $y = f(-x)$ in the y -axis. Invariant points lie on the mirror axis.

Show your work on separate paper.

Core Skills (K)

Translations · Reflections · Stretches · Mapping Notation · Equations from Graphs

K1 Translations

The table lists four points on the graph of $y = f(x)$. For each function, give the images of the four points.

x	-2	0	1	4
$f(x)$	3	-1	0	2

- $y = f(x) + 4$
- $y = f(x + 3)$
- $y = f(x - 2) - 5$

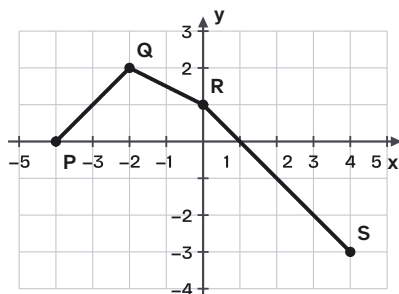
K2 Reflections

Let $f(x) = \sqrt{x + 4} - 1$.

- Write the equation of $y = -f(x)$ and state its range in interval notation.
- Write the equation of $y = f(-x)$ and state its domain in interval notation.
- State the invariant points of $y = -f(x)$ and of $y = f(-x)$.

K3 Stretches & Compressions

The graph of $y = f(x)$ is shown. Each grid square is 1 unit.



- State the images of P , Q , R and S on $y = 2f(x)$.
- State the images of P , Q , R and S on $y = f(2x)$.

Mapping rule for $y = af[k(x - d)] + c$: $(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$.
Stretches and reflections first, translations last.

Factor first: $f(2x - 6) = f(2(x - 3))$ is a shift of 3 units right, not 6.

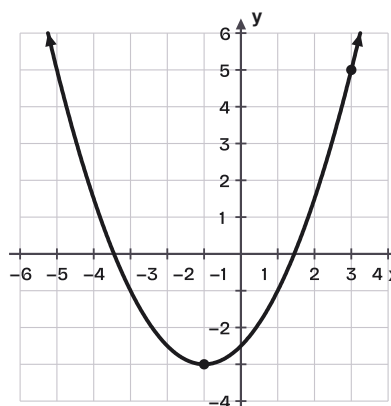
- State the invariant points of $y = 2f(x)$ and of $y = f(2x)$.

K4 Mapping Notation

- Write the mapping rule for $y = -2f(3(x - 4)) + 5$.
- The point $(-3, 2)$ is on $y = f(x)$. Find its image on $y = -2f(3(x - 4)) + 5$.
- A function f has domain $[-4, 8]$ and range $[-3, 5]$. State the domain and range of $y = -2f(2x + 6) + 1$ in interval notation.

K5 Reading Equations & Graphs

- State the transformations, in order, that map $y = f(x)$ onto $y = \frac{1}{2}f(-3x + 9) - 2$.
- The graph is a transformation of $y = x^2$. Determine its equation. Each grid square is 1 unit.



- For $y = -\frac{2}{x + 4} + 1$, state the equations of the asymptotes and the image of the point $(1, 1)$ on $y = \frac{1}{x}$.

Real World (A)

Translations in Time · Stretches in Context · Square-Root Models

A1 Second Kick

Priya kicks a ball from the ground. Its height, in metres, after t seconds is $h(t) = -5t^2 + 20t$. Her friend kicks an identical ball the same way from the same spot, 1.5 s later. Let $g(t)$ be the height of the friend's ball, with t still measured from Priya's kick.

- Write $g(t)$ in terms of h , then in vertex form.
- When does each ball reach its maximum height, and when does each land?
- State the domain of g in this context. The formula gives $g(1) = -11.25$. Where is the friend's ball at $t = 1$?

A2 Fair Ferris Wheel

At a summer fair in Ottawa, a Ferris wheel seat starts at its lowest point and makes one turn in 40 s. The table gives its height $h(t)$, in metres, after t seconds.

t (s)	0	10	20	30	40
$h(t)$ (m)	2	10	18	10	2

- The operator speeds the wheel up so one turn takes 30 s. Write the new height function in terms of h , and find when a rider first reaches the top.
- A bigger wheel turns in 40 s, and every height is 1.5 times the height in the table. Write its height function in terms of h and state its range.
- Riders board from a platform 2 m high, at the seat's lowest point. Which of the two new rides can still use this platform? Explain.

A3 Horizon from the Cliff

On a clear day, the distance to the horizon, in kilometres, from eyes h metres above the water is $d(h) = 3.57\sqrt{h}$. A lighthouse on Lake Superior stands on a cliff 45 m above the water, and its tower is 36 m tall. Let $D(h)$ be the horizon distance for a visitor whose eyes are h metres above the cliff top.

- Write $D(h)$ in terms of d , then as an equation. Which transformations of $y = \sqrt{h}$ give D ?
- Visitors stand anywhere from the cliff top to the top of the tower. State the domain and range of D in this context, to one decimal place.
- A guide wants to know the height at which the horizon is exactly 20 km away. Solve $D(h) = 20$ and interpret the result.

Think It Through (T)

Error Analysis · Counterexamples

T1 Order Matters

A student wants the graph of $y = \sqrt{x}$ compressed horizontally by a factor of $\frac{1}{3}$, then translated 2 units right. They write $y = \sqrt{3x - 2}$.

- Find the endpoint of the student's graph and the endpoint the description requires.
- Write the correct equation, both factored and expanded under the root.
- The student's equation does match a translation and a compression of $y = \sqrt{x}$, in some order. Name them in the order applied.

T2 Do the Negatives Cancel?

A student makes two claims.

Claim 1: " $y = -f(-x)$ is always the same graph as $y = f(x)$, because the two negatives cancel."

Claim 2: " $g(x) = x^3 - 2x + 1$ is odd, because every power of x is odd."

- Use $f(x) = (x - 1)^2$ to show Claim 1 is false: write $-f(-x)$ and give a point on $y = f(x)$ that is not on $y = -f(-x)$.
- Describe the functions for which Claim 1 is true, and give one example other than $y = x$.
- Evaluate $g(-1)$ and $-g(1)$. Is g even, odd, or neither?

Boss Level

Optional challenge

Mirror and Slide

Let R be a reflection in the y -axis and S a translation 4 units right. Transformations are applied to the graph of $y = f(x)$ one after another, from left to right.

- Apply R , then S , then R . Write the final equation in terms of f , and state the single transformation it equals.
- The point $(1, 6)$ is on $y = f(x)$. Apply the pair " R then S " 50 times in a row. Where does the point end up?
- Find every point that one pair " R then S " leaves where it is.



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