

Trigonometric Ratios

Ontario MCR3U • right triangles, angles to 360° , identities, the sine and cosine laws

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Worksheet, answers &
this cheat sheet



MOST TESTED Given $\cos \theta = -0.6$, find all θ from 0° to 360° . (card 4)

Which tool?

There is a right
angle

SOH CAH TOA

Two sides and the
angle **between**, or
three sides

Cosine law

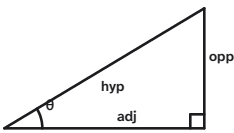
A side and its **opposite**
angle

Sine law

1 Right triangles

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

USE Stand at θ : **opposite** is the side that doesn't touch it. Missing angle: use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.



TIP Angles of elevation and depression are measured from the **horizontal**; bearings, clockwise from **north**.

TRAP $\sin 35^\circ = 8/x$ gives $x = 8/\sin 35^\circ \approx 13.9$, not $8 \sin 35^\circ \approx 4.6$: the hypotenuse is the longest side.

2 Special angles

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef.

TIP The sin row is $\sqrt{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}$, each over 2. The cos row is the same, backwards.

3 Any angle to 360°

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

USE $P(x, y)$ on the terminal arm, $r = \sqrt{x^2 + y^2}$. Coterminal angles: $\theta \pm 360^\circ$.

S sin + $\beta = 180^\circ - \theta$	A all + $\beta = \theta$
T tan + $\beta = \theta - 180^\circ$	C cos + $\beta = 360^\circ - \theta$

EXAMPLE $\sin 240^\circ$: QIII, $\beta = 60^\circ$, sin negative, so $-\sqrt{3}/2$.

TRAP The related acute angle (reference angle) is measured to the x -axis, **never** the y -axis: for 240° it is 60° , not 30° .

4 Find θ from a ratio

USE 1. Find β from the **positive** value. 2. CAST: the two quadrants with the right sign. 3. Write θ in each.

EXAMPLE $\cos \theta = -0.6$: $\beta = \cos^{-1} 0.6 \approx 53.1^\circ$; cos is negative in QII and QIII, so $\theta \approx 126.9^\circ$ or 233.1° .

TRAP The calculator gives one angle, but there are usually **two**.

5 Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

USE To prove: start from the more complex side, rewrite it in sin and cos, and work on that side only.

EXAMPLE $\sin \theta = -5/13$ in QIV: $\cos^2 \theta = 144/169$ and cos is positive in QIV, so $\cos \theta = 12/13$.

TRAP $\sec 60^\circ = 1 \div \cos 60^\circ = 1 \div 0.5 = 2$, not 0.5.

6 Sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

USE AAS, SSA: a side and its **opposite** angle. ASA: find the third angle first.

TRAP Each side pairs with the angle **across** from it, never an angle next to it.

7 The ambiguous case (SSA)

Given $\angle A$, a , b . Acute $\angle A$: $h = b \sin A$.

$a < h$	no triangle
$a = h$	one (right) triangle
$h < a < b$	two triangles
$a \geq b$	one triangle
obtuse $\angle A$, $a \leq b$	no triangle
obtuse $\angle A$, $a > b$	one triangle

EXAMPLE $\angle A = 30^\circ$, $a = 6$, $b = 10$: $h = 5 < 6 < 10$: $\angle B \approx 56.4^\circ$ or 123.6° .

TRAP $\sin^{-1}$ gives only the acute angle. In SSA, test $180^\circ - \theta$ too.

8 Cosine law

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

USE SAS: the third side. SSS: an angle; find the **largest** first, so the other two are acute.

EXAMPLE Sides 5, 7, 8: $\cos C = (25 + 49 - 64)/70 = 1/7$, so $C \approx 81.8^\circ$.

TRAP $c^2 = 8^2 + 10^2 - 2(8)(10) \cos 60^\circ = 84$, not $(164 - 160) \cos 60^\circ$: multiply first.

9 3D problems

USE Redraw each triangle **flat**; a tower meets level ground at 90° . Solve the triangle you know most about, then carry the **shared side** across. Base not right-angled? Sine or cosine law.

TRAP A negative h^2 means the wrong hypotenuse. Redraw; don't take the absolute value.