

# Trigonometric Ratios Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 50 min

**Recommended:** K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



vertexmath.ca  
Hints, answers &  
more worksheets

## QUICK REFERENCE

**Right triangles:** opposite and adjacent depend on which angle you stand at. The hypotenuse is always across from the  $90^\circ$  angle.

**Identities:**  $\sin^2 \theta + \cos^2 \theta = 1$ ,  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ ,  $\csc \theta = \frac{1}{\sin \theta}$ ,

$\sec \theta = \frac{1}{\cos \theta}$ ,  $\cot \theta = \frac{1}{\tan \theta}$ .

$\sin^{-1}$  only returns an acute angle, but  $\sin \theta = \sin(180^\circ - \theta)$ . Always check whether the obtuse angle works too.

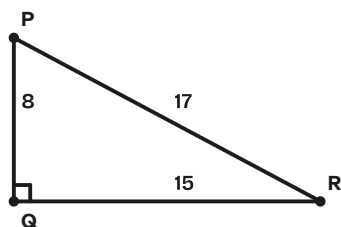
Show your work on separate paper.

## Core Skills (K)

Right Triangles · Any Angle · Identities · Sine Law · Cosine Law

### K1 Right-Triangle Ratios

Parts (a) and (b) use right triangle  $PQR$  below, with  $\angle Q = 90^\circ$ .



- State  $\sin P$ ,  $\cos P$  and  $\tan P$  as fractions.
- Determine  $\angle P$  to the nearest tenth of a degree.
- In a different right triangle  $ABC$ ,  $\angle B = 90^\circ$ ,  $\angle A = 35^\circ$  and  $BC = 8$  cm. Determine  $AC$  to one decimal place.

### K2 Any Angle

- Complete the table: state the reference angle of each angle and the sign of  $\tan \theta$ .

| $\theta$              | $110^\circ$ | $250^\circ$ | $335^\circ$ |
|-----------------------|-------------|-------------|-------------|
| reference angle       |             |             |             |
| sign of $\tan \theta$ |             |             |             |

- Determine the exact values of  $\sin 240^\circ$ ,  $\cos 240^\circ$  and  $\tan 240^\circ$ .
- Determine the exact value of  $\sin^2 120^\circ + \cos 225^\circ \tan 135^\circ$ .

**CAST:** All ratios are positive in Quadrant I, Sine in II, Tangent in III, Cosine in IV. A reference angle is always measured to the  $x$ -axis.

**Sine law**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$  for AAS, ASA, SSA. **Cosine law**  $c^2 = a^2 + b^2 - 2ab \cos C$  for SAS, SSS.

### K3 Identities

Given  $\sin \theta = -\frac{8}{17}$  and  $270^\circ < \theta < 360^\circ$ .

- Determine the exact value of  $\cos \theta$ .
- Determine the exact values of  $\tan \theta$  and  $\csc \theta$ .
- Simplify  $\frac{(\sin \theta + \cos \theta)^2 - 1}{\cos^2 \theta}$ , then evaluate it for this  $\theta$ .

### K4 Sine Law

In  $\triangle DEF$ ,  $\angle D = 52^\circ$ ,  $\angle E = 71^\circ$  and  $f = 15$  cm. Give lengths to one decimal place.

- Determine  $d$ .
- Determine  $e$ .
- In  $\triangle XYZ$ ,  $x = 11$  cm,  $y = 8$  cm and  $\angle X = 70^\circ$ . Determine  $\angle Y$  to the nearest tenth of a degree.

### K5 Cosine Law

Give lengths to one decimal place and angles to the nearest tenth of a degree.

- In  $\triangle ABC$ ,  $b = 7$  cm,  $c = 10$  cm and  $\angle A = 48^\circ$ . Determine  $a$ .
- In  $\triangle KLM$ ,  $k = 5$  m,  $l = 8$  m and  $m = 11$  m. Determine  $\angle M$ .
- In  $\triangle STU$ ,  $s = 6$  cm,  $t = 9$  cm and  $\angle U = 125^\circ$ . Determine  $u$ .

## Real World (A)

Ambiguous Case · Cosine Law · 3D Problems

### A1 Signal on the Highway

A straight highway in Northern Ontario starts at a town  $A$ . A cell tower  $T$  is 12 km from  $A$ , and segment  $AT$  makes an angle of  $32^\circ$  with the highway. The tower's signal reaches 9 km in every direction.

- a) A car is at a point  $C$  on the highway, exactly 9 km from the tower. Use the sine law to find the two possible measures of  $\angle ACT$ , to the nearest tenth of a degree.
- b) How far from  $A$  does a car driving away from town gain signal, and how many kilometres does it then drive with signal?
- c) To cut costs, the provider installs an antenna that reaches only 6 km. Repeat the sine-law step from (a). What does the result mean for drivers on this highway?

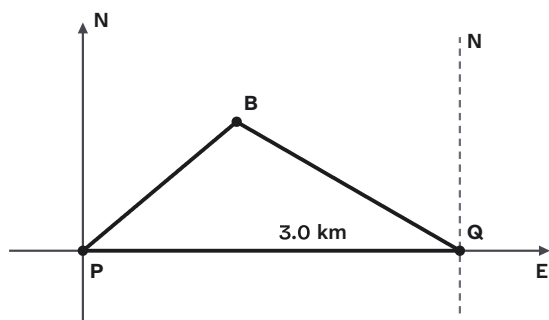
## A2 Flower Bed

A landscaper in Kitchener is building a triangular flower bed with edges 4.0 m, 5.5 m and 8.0 m long.

- a) Determine the largest angle of the bed, to the nearest tenth of a degree.
- b) A tree goes in the corner between the 4.0 m and 8.0 m edges. Determine the angle of that corner.
- c) A client asks for a second bed with edges 3.0 m, 4.5 m and 8.0 m. Use the cosine law to find its largest angle. What should the landscaper tell the client?

## A3 Balloon Spotting

Two observers stand on level farmland at  $P$  and  $Q$ , with  $Q$  3.0 km due east of  $P$ . A hot-air balloon hovers directly above point  $B$  on the ground. From  $P$ ,  $B$  is on a bearing of  $050^\circ$ ; from  $Q$ ,  $B$  is on a bearing of  $300^\circ$ . From  $P$ , the angle of elevation of the balloon is  $20^\circ$ .



Top view of the ground

- a) Determine the three interior angles of  $\triangle PQB$  on the ground.
- b) Determine the height of the balloon, to the nearest metre.
- c) The observer at  $P$  has a laser rangefinder that measures straight-line distances up to 1.65 km. Can it measure the distance to the balloon? Explain.

## Think It Through (T)

Error Analysis · Explain Why

### T1 Proof or Not?

A student wants to prove  $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$  and writes:

"Cross-multiply:

$\cos^2 x = (1 - \sin x)(1 + \sin x) = 1 - \sin^2 x$ . That is the Pythagorean identity, which is true, so the identity is proved."

- a) Explain why this is not a valid proof.
- b) Write a valid proof, working on the left side only.
- c) A second student claims  $\sin x + \cos x = 1$  is an identity because it works for  $x = 0^\circ$  and  $x = 90^\circ$ . Find a counterexample.

### T2 The Hidden Angle

A student solves  $\triangle ABC$  with  $a = 6$ ,  $b = 9$  and  $c = 13$ .

They use the cosine law to get  $\angle A \approx 23.9^\circ$ , then the sine law to get  $\angle C \approx 61.2^\circ$ , and finally  $\angle B \approx 180^\circ - 23.9^\circ - 61.2^\circ = 94.9^\circ$ .

- a) Use the cosine law to find  $\angle C$ . Which of the student's angles are wrong?
- b) Explain why the sine law gave  $61.2^\circ$ .
- c) Keep  $a = 6$  and  $b = 9$ . Find all lengths  $c$  for which  $\angle C$  is obtuse.

## Boss Level

Optional challenge

### Tower Walk

Point  $A$  is on level ground, due south of the base of a vertical tower of height  $h$  metres. From  $A$ , the angle of elevation of the top is  $45^\circ$ . A hiker walks 100 m due east from  $A$  to  $B$ , where the angle of elevation of the top is  $30^\circ$ .

- a) Write the distances from  $A$  and from  $B$  to the base of the tower in terms of  $h$ , exactly.
- b) Find  $h$  exactly.
- c) Find the angle of elevation of the top from  $M$ , the midpoint of  $AB$ , to the nearest tenth of a degree.



vertexmath.ca  
Hints, answers &  
more worksheets

Free to print, copy, adapt and share for  
classroom use · CC BY-NC 4.0 · ©  
VertexMath