

Combining Functions

Ontario MHF4U • sums, products, quotients, composition, equations, inequalities and models

MOST TESTED For $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 2$, find $f(g(x))$ and $g(f(x))$ with their domains. (card 4)

vertexmath.ca
Worksheet, answers &
this cheat sheet



Which combination?

Quantities add or subtract

$f \pm g$

One quantity scales another

fg

One output is the next input

$f(g(x))$

1 Sums and differences

$$(f \pm g)(x) = f(x) \pm g(x)$$

USE Domain: where **both** f and g are defined. Graph: add (or subtract) the y -values at each x .

EXAMPLE $f(x) = \sqrt{x}$, $g(x) = \sqrt{4-x}$:
 $f + g$ has domain $[0, 4]$.

TRAP Subtract **every** term of g :

$$(x^2 + 1) - (2x - 3) = x^2 - 2x + 4, \text{ not } x^2 - 2x - 2.$$

2 Products and quotients

$$(fg)(x) = f(x)g(x) \quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

USE Domain of f/g : both defined **and** $g(x) \neq 0$, found before simplifying. Sign of fg : multiply the signs. $y = ae^{-kx} \sin x$: the amplitude ae^{-kx} shrinks toward 0 but never reaches it.

TRAP $(x^2 - 9)/(x + 3)$ simplifies to $x - 3$ but still excludes $x = -3$.

3 Composition

$$(f \circ g)(x) = f(g(x))$$

USE Inside first: apply g , then f . Replace **every** x in f with $g(x)$. In general $f(g(x)) \neq g(f(x))$, and $f \circ g$ is not fg .

EXAMPLE $f(x) = x^2$, $g(x) = x + 3$:
 $f(g(x)) = (x + 3)^2$, but
 $g(f(x)) = x^2 + 3$.

TRAP $f(g(2))$: find $g(2)$ first, then put it into f . Not $f(2)$ combined with $g(2)$.

4 Composite domain and decomposition

USE Domain of $f \circ g$: x in the domain of g **and** $g(x)$ in the domain of f . Find it before simplifying. To decompose, name the inner step g and the outer step f .

EXAMPLE $f(x) = \sqrt{x-1}$,
 $g(x) = x^2 + 2$: $f(g(x)) = \sqrt{x^2 + 1}$ on $\mathbb{R}$;
 $g(f(x)) = x + 1$ on $x \geq 1$.

TRAP $p(x) = x^2$, $q(x) = \sqrt{x-2}$:
 $p(q(x)) = x - 2$, but its domain is $x \geq 2$, not $\mathbb{R}$.

5 Equations and inequalities

USE $f(x) = g(x)$: solve $f(x) - g(x) = 0$, or read the intersections from a graph. Inequality: everything on one side, factor, test every interval. Squaring both sides can add roots: check them.

TRAP A squared factor never changes sign: $(x + 1)^2(x - 3) < 0$ means $x < 3$, $x \neq -1$.

TRAP Never divide by x of unknown sign: $x^2 > 2x$ is $x < 0$ or $x > 2$.

6 Modelling

USE Profit = revenue – cost: a difference. Price $\times$ quantity: a product. One quantity feeds the next: a composition, inner first. Check the domain against the context.

EXAMPLE Fuel $V(d) = 0.08d$ L and cost
 $C(V) = 1.6V$: cost of a trip
 $C(V(d)) = 0.128d$.

TRAP Use the vertex only if it is in the allowed domain, such as below a capacity limit.