

Combining Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Sum, difference, product, quotient: $(f \pm g)(x) = f(x) \pm g(x)$ and $(fg)(x) = f(x)g(x)$. The domain is where **both** f and g are defined; for $\frac{f}{g}$, also remove every x with $g(x) = 0$.

Domain of $f \circ g$: x must be in the domain of g and $g(x)$ must be in the domain of f . Find it before you simplify.

Squaring both sides can create **extraneous roots**: check every solution in the original equation.

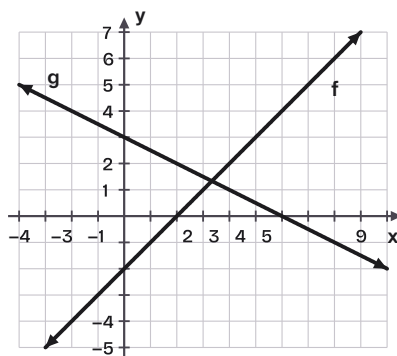
Show your work on separate paper.

Core Skills (K)

Sums & Differences · Products & Quotients · Composition · Solving

K1 Sums & Differences

- For $f(x) = 3x^2 - 2x + 5$ and $g(x) = x^2 - 4x + 7$, simplify $(f - g)(x)$.
- For $p(x) = \sqrt{x+2}$ and $q(x) = \sqrt{4-x}$, state the domain of $(p + q)(x)$ in interval notation.
- The lines $y = f(x)$ and $y = g(x)$ are graphed below. Each grid square is 1 unit. Evaluate $(f + g)(4)$, then find the value of x where $(f + g)(x) = 0$.



K2 Products & Quotients

- For $f(x) = x + 4$ and $g(x) = 2x - 3$, simplify $(fg)(x)$.
- For $f(x) = x^2 - 9$ and $g(x) = x^2 + x - 6$, simplify $\left(\frac{f}{g}\right)(x)$ and state its domain.
- For $u(x) = \sqrt{x-1}$ and $v(x) = x - 4$, state the domains of $\left(\frac{u}{v}\right)(x)$ and $\left(\frac{v}{u}\right)(x)$ in interval notation.

Composition: $(f \circ g)(x) = f(g(x))$: apply g first, then f . In general $f(g(x)) \neq g(f(x))$, and $f \circ g$ is not the product fg .

Inequalities: move everything to one side, factor, and test every interval. A squared factor does not change sign, and never divide by an expression whose sign you don't know.

K3 Composition from a Table

Use the table of values for f and g .

x	-2	-1	0	1	2	3
$f(x)$	3	0	-1	0	3	8
$g(x)$	2	3	1	-2	0	-1

- Evaluate $f(g(0))$ and $g(f(0))$.
- Evaluate $(g \circ g)(1)$ and $(f \circ g)(2)$.
- Find every x in the table for which $f(g(x)) = 0$.

K4 Composition Rules

- For $f(x) = x^2 + x$ and $g(x) = x - 2$, simplify $f(g(x))$.
- For the same f and g , simplify $g(f(x))$.
- For $p(x) = x^2 - 5$ and $q(x) = \sqrt{x+3}$, simplify $p(q(x))$ and state its domain in interval notation.

K5 Solving with Combined Functions

- Find the points of intersection of $f(x) = x^2 - 2x$ and $g(x) = x + 4$.
- For $p(x) = \sqrt{x+6}$ and $q(x) = x$, solve $p(x) = q(x)$.
- For $f(x) = (x+1)^2$ and $g(x) = x - 3$, solve $(fg)(x) > 0$. Give your answer in interval notation.

Real World (A)

Profit Models · Damped Motion · Composite Models

A1 Festival Food Truck

A food truck at a summer festival in Ottawa sells x meals a day when it charges $p(x) = 15 - 0.025x$ dollars per meal. Its daily cost is $C(x) = 4x + 400$ dollars.

- Write the revenue $R(x) = x \cdot p(x)$ and the profit $P(x) = R(x) - C(x)$ in simplified form.

- b) How many meals must the truck sell to break even? For which numbers of meals does it make a profit?
- c) The truck's fridge holds ingredients for only 150 meals a day. How many meals should it sell to make the most profit? State the price per meal and the daily profit.

A2 Diving Board

After a diver leaves a diving board, the vertical position of the board's tip, in centimetres above its rest position, is modelled by $d(t) = 12(0.5)^t \cos(2\pi t)$, where t is the time in seconds.

- a) Evaluate $d(0.5)$ and $d(1)$, to one decimal place. Where is the tip at each time?
- b) The factor $12(0.5)^t$ sets the size of each swing. After how many seconds are the swings smaller than 0.5 cm? Round to one decimal place.
- c) Evaluate the swing size $12(0.5)^t$ at $t = 10$. According to the model, does the board ever stop moving? Is the model still useful at $t = 10$?

A3 Weather Balloon

A weather balloon rises so that its altitude, in kilometres, t minutes after launch is $h(t) = 0.3t$. Up to an altitude of 11 km, the air temperature in $^{\circ}\text{C}$ at altitude h km is $T(h) = 15 - 6.5h$.

- a) Find $T(h(t))$ in simplified form. What do its input and its output represent?
- b) When does the balloon reach air at -24°C , and at what altitude?
- c) Evaluate $T(h(50))$. Should you believe the result? State the domain of $T \circ h$ in this context.

Think It Through (T)

Error Analysis · Building Functions

T1 Spot the Flaw

A student solves $f(x) \geq g(x)$ for $f(x) = x^3$ and $g(x) = 4x$:

" $x^3 \geq 4x$. Divide both sides by x : $x^2 \geq 4$. Therefore $x \leq -2$ or $x \geq 2$."

- a) Test $x = -1$ and $x = -3$ in the original inequality. What does each test show about the student's answer?
- b) Identify the flawed step and explain what went wrong.
- c) Solve $x^3 \geq 4x$ correctly. Give your answer in interval notation.

T2 Build It Backwards

Many answers are possible. Neither f nor g may be $y = x$.

- a) Find f and g such that $f(g(x)) = (3x - 1)^5 + 2$.
- b) Write $k(x) = \sqrt{x^2 + 1}$ as $f(g(x))$ in two different ways.

- c) Find two functions f and g with $f(g(x)) = g(f(x))$ for every x . Show that they work.

Boss Level

Optional challenge

Going in Circles

Let $f(x) = \frac{1}{1-x}$. Define $f_1(x) = f(x)$ and $f_{n+1}(x) = f(f_n(x))$, so f_n applies f a total of n times.

- a) Simplify $f_2(x) = f(f(x))$.
- b) Simplify $f_3(x)$.
- c) Find $f_{2026}(2)$.



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