

Exponential and Logarithmic Functions

Ontario MHF4U • exponential and log graphs, transformations, laws, equations and models

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Worksheet, answers &
this cheat sheet



MOST TESTED Solve $\log_2 x + \log_2(x - 2) = 3$. (card 7)

Which strategy?

Both sides
share a base

Equate exponents

Bases don't match

Take logs

Logs on one
side

Condense, convert

1 Exponential functions

USE $y = b^x$, $b > 0$, $b \neq 1$: domain $\mathbb{R}$, range $y > 0$, asymptote $y = 0$, y -intercept 1. $b > 1$ grows, $0 < b < 1$ decays. A table has constant **ratios**.

TRAP $2^{-3} = 1/8$, still positive: the range is $y > 0$ and the curve never reaches $y = 0$.

2 Exponential transformations

$$y = a b^{k(x-d)} + c: \text{asymptote } y = c$$

USE Factor k out of the exponent first. Map $(x, y) \rightarrow (x/k + d, ay + c)$.

TRAP $2^{2x-6} = 2^{2(x-3)}$: the shift is 3 right, not 6.

3 Equal bases

USE Write both sides as powers of the same base, then set the exponents equal. A difference like $b^{x+3} - b^x$: factor out b^x .

EXAMPLE $4^{x+1} = 8^x$: $2^{2x+2} = 2^{3x}$, so $x = 2$.

TRAP $2^{x+3} - 2^x = 2^x(2^3 - 1) = 7 \cdot 2^x$, not 2^3 .

4 Logarithms

$$\log_b x = y \iff b^y = x$$

USE $y = \log_b x$: domain $x > 0$, asymptote $x = 0$, x -intercept 1. It is the inverse of b^x . log means base 10; ln means base e .

TRAP $\log_x 125 = 3$ means $x^3 = 125$, so $x = 5$, not 125^3 .

5 Log transformations

$$y = a \log_b[k(x-d)] + c: \text{asymptote } x = d$$

USE Factor k out first. Domain: solve $k(x-d) > 0$. The x -intercept comes from $a \log_b[k(x-d)] + c = 0$.

TRAP $y = \log(5-x)$ needs $5-x > 0$: domain $x < 5$, not $x > 5$.

6 Laws of logarithms

$$\log_b(mn) = \log_b m + \log_b n$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

$$\log_b m^n = n \log_b m \quad \log_b m = \frac{\log m}{\log b}$$

TRAP $\log u + \log v = \log(uv)$, never $\log(u+v)$.

TRAP $\log_2 64 \div \log_2 8 = 6 \div 3 = 2$: a quotient of logs is not the log of a quotient.

7 Solving with logarithms

USE Exponential: isolate the power, then take log of both sides. Log equation: condense, convert to exponential form, then check every root; each argument must be positive.

EXAMPLE $\log_2 x + \log_2(x-2) = 3$: $x(x-2) = 8$, so $x = 4$ ($x = -2$ is rejected).

TRAP Isolate first: $20(1.05)^t = 50$ becomes $1.05^t = 2.5$, so $t = \log 2.5 / \log 1.05 \approx 18.8$.

8 Models and log scales

$$A = A_0(1+r)^t \quad A = A_0(2)^{t/d}$$

$$A = A_0 \left(\frac{1}{2} \right)^{t/h}$$

USE t , d and h in the same units. Log scales: +1 in pH or magnitude, or +10 dB, means $\times 10$. Instantaneous rate: shrink the interval.

TRAP 96 dB is $10^{3.6} \approx 3981$ times as intense as 60 dB, not 1.6 times.