

Exponential and Logarithmic Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Transformations of $y = a \cdot b^{k(x-d)} + c$: factor k out of the exponent before reading d . The asymptote is $y = c$.

Laws: $\log_b(mn) = \log_b m + \log_b n$, $\log_b \frac{m}{n} = \log_b m - \log_b n$,

$\log_b m^n = n \log_b m$, $\log_b m = \frac{\log m}{\log b}$.

$\log(a+b) \neq \log a + \log b$. After solving a log equation, check every root in the **original** equation: each argument must be positive.

Show your work on separate paper.

Core Skills (K)

Exponential Graphs · Transformations · Equivalent Bases · Evaluating Logs · Log Laws

K1 From a Table

The table shows values of an exponential function

$$f(x) = a \cdot b^x.$$

x	1	2	3	4
$f(x)$	20	5	1.25	0.3125

- Determine a and b .
- State whether f shows growth or decay. Then state its domain, its range and the equation of its asymptote.
- Evaluate $f(-2)$ and $f\left(\frac{1}{2}\right)$.

K2 Transformed Exponential

$$\text{Let } g(x) = -3(2)^{2x-6} + 4.$$

- Write the exponent in the form $k(x-d)$. Then state a , k , d and c .
- State the equation of the asymptote, the domain and the range of g .
- The point $(2, 4)$ lies on $y = 2^x$. Determine its image on g . Then determine the y -intercept of g .

K3 Same Base

Solve each equation by writing both sides as powers of the same base.

- $9^{x+1} = 27^{x-1}$
- $4^x = \frac{1}{8\sqrt{2}}$
- $2^{x+3} - 2^x = 56$

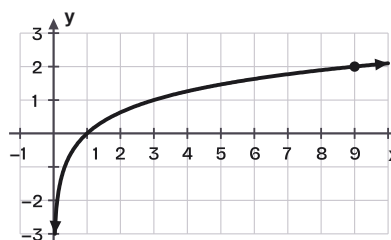
K4 Evaluating Logarithms

- Evaluate $\log_2 \frac{1}{32}$, $\log_8 4$ and $\ln \sqrt{e}$.

Definition: $\log_b x = y \Leftrightarrow b^y = x$. The argument must be positive, so $y = \log_b x$ has domain $(0, \infty)$ and asymptote $x = 0$.

Models: growth of $r\%$ per period uses the base $1 + \frac{r}{100}$. A half-life h gives $\left(\frac{1}{2}\right)^{t/h}$, with t and h in the same units.

- Solve $\log_x 125 = 3$ and $\log_6 x = -2$.
- The graph of $y = \log_b x$ passes through the marked point. Determine b , then state the domain and range of $y = b^x$. Each grid square is 1 unit.



K5 Laws of Logarithms

- Evaluate $\log_6 4 + \log_6 9$ and $\log_3 54 - \log_3 2$.
- Write as a single logarithm: $2 \log x + \frac{1}{2} \log y - 3 \log z$.
- Evaluate $\frac{\log_2 64}{\log_2 8}$ and $\log_2 64 - \log_2 8$.

Real World (A)

Growth Models · Log Scales · Solving in Context

A1 Petri Dish

A lab culture starts with 500 bacteria and doubles every 20 minutes.

- Write $N(t)$, the number of bacteria after t hours.
- Find the number of bacteria after 4 hours.
- The dish can hold about 10^9 bacteria. When does the model reach this? Then evaluate $N(24)$ and say what it tells you about the model.

A2 Leaf Blowers

The sound level, in decibels, of a sound with intensity I (in W/m^2) is $L = 10 \log \left(\frac{I}{I_0} \right)$, where $I_0 = 10^{-12} \text{ W/m}^2$.

- a) A conversation has intensity 10^{-6} W/m^2 . Find its sound level.
- b) A leaf blower measures 96 dB. How many times as intense as the conversation is it?
- c) Two identical 96 dB leaf blowers run side by side. Their intensities add. Find the combined sound level.

A3 Cooling Tea

A cup of tea is left on a counter in a 20°C room. Its temperature, in $^\circ\text{C}$, after t minutes is

$$T(t) = 20 + 70(0.5)^{t/12}.$$

- a) Find the temperature of the tea after 30 minutes.
- b) When is the tea at 50°C ?
- c) When does the tea reach 18°C ? Interpret your result.

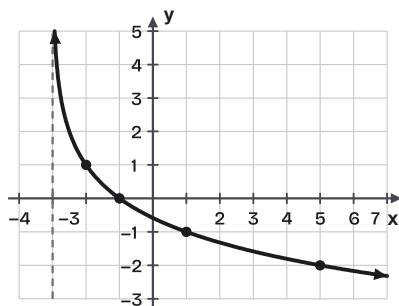
Think It Through (T)

Building Log Functions · Error Analysis

T1 Build the Log

Each part asks for the equation of a transformed logarithmic function.

- a) The graph is a transformation of $y = \log_2 x$. Determine its equation in the form $y = a \log_2(x - d) + c$. Each grid square is 1 unit.



- b) Write the equation of a logarithmic function with vertical asymptote $x = 5$, domain $(-\infty, 5)$ and x -intercept 4.
- c) A function $y = a \log_3(x - d) + c$ has vertical asymptote $x = -2$ and passes through $(1, 5)$ and $(7, 7)$. Determine a , d and c .

T2 Spot the Flaw

A student solves $\log_2(x + 2) + \log_2(x - 4) = 4$:

"Product law: $\log_2((x + 2)(x - 4)) = 4$, so

$(x + 2)(x - 4) = 16$. Then $x^2 - 2x - 24 = 0$ and

$(x - 6)(x + 4) = 0$. Both $x = 6$ and $x = -4$ are solutions."

- a) Check each claimed solution in the original equation. Which one fails, and why?
- b) State the domain of the original equation and the domain of $\log_2((x + 2)(x - 4)) = 4$. Explain how this lets the wrong root in.

- c) Another student writes $\log_2(x + 2) + \log_2(x - 4) = \log_2(2x - 2)$. Use $x = 6$ to show this is false in general. Then find the only x for which it is true.

Boss Level

Optional challenge

A Chain of Logs

Use the change of base formula throughout.

- a) Evaluate $\log_2 3 \cdot \log_3 4$.
- b) Evaluate $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_{63} 64$.
- c) Solve $\log_2 x + \log_4 x + \log_{16} x = 7$.



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