

Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Even: $f(-x) = f(x)$, symmetric in the y -axis. **Odd:**

$f(-x) = -f(x)$, symmetric about the origin. Test with $f(-x)$, not by looking at exponents.

Inverse: swap x and y , then solve for y . The domain of f^{-1} is the range of f .

In $f(x+h) - f(x)$, the minus sign applies to **every** term of $f(x)$.

Show your work on separate paper.

Core Skills (K)

Domain · Transformations · Piecewise · Inverses · Rates of Change

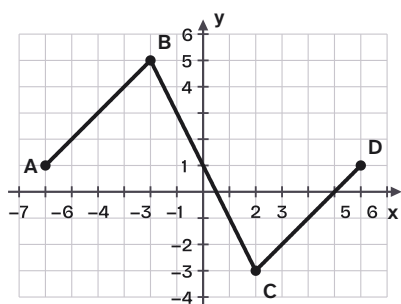
K1 Domain First

Let $f(x) = \frac{x^2 - x - 6}{x - 3}$.

- State the domain of f in interval notation.
- Simplify $f(x)$. Then evaluate $f(1)$ and $f(3)$, if they exist.
- State the domain of $g(x) = \frac{\sqrt{8-2x}}{x+1}$ in interval notation.

K2 Where Does the Point Go?

The graph of $y = f(x)$ is shown. Each grid square is 1 unit. Find the image of the given point on the graph of each new function.



- Point B , for $y = f(x+3) - 2$.
- Point C , for $y = -2f(-x)$.
- Point D , for $y = 3f(2x+12) - 1$.

Transformations: in $y = af(k(x-d)) + c$, factor k out first. $y = f(2x+8) = f(2(x+4))$ shifts the graph 4 units left, not 8.

Rate of change: the average rate on $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$. The instantaneous rate is the value $\frac{f(a+h) - f(a)}{h}$ approaches as $h \rightarrow 0$.

K3 Piecewise Functions

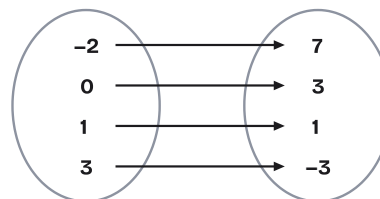
Let

$$f(x) = \begin{cases} x^2 + 2, & x < 1 \\ 1 - x, & x \geq 1 \end{cases}$$

- Evaluate $f(-2)$, $f(1)$ and $f(4)$.
- State the domain and range of f in interval notation.
- Determine whether f is continuous at $x = 1$. If it is not, find the size of the jump.

K4 Inverses

- The mapping diagram shows a function f . Evaluate $f^{-1}(-3)$ and $f^{-1}(7)$.



- Determine $g^{-1}(x)$ for $g(x) = 2(x-1)^3 + 5$.
- Determine $h^{-1}(x)$ for $h(x) = 3 - \sqrt{x+2}$, and state its domain and range.

K5 Rates of Change

Let $f(x) = x^2 - 3x + 4$.

- Find the average rate of change of f on $[1, 4]$.
- Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$, for $h \neq 0$.
- Use (b) to find the instantaneous rate of change of f at $x = 2$.

Real World (A)

Rates in Context · Inverses in Context · Piecewise Pricing

A1 Battery Drain

Priya records her phone's battery level B , in %, every 2 hours during a long day. t is the time in hours.

t	0	2	4	6	8
B	100	84	64	42	22

- Find the average rate of change of B from $t = 0$ to $t = 4$, with units. What does it mean for the phone?
- Over which 2-hour interval did the battery drain fastest? Give the rate.
- Use the rate from $t = 6$ to $t = 8$ to predict the battery level at $t = 11$. Does the prediction make sense?

A2 Skid Marks

On a wet road, the braking distance of a car, in metres, at a speed of v km/h is modelled by $d(v) = \frac{v^2}{200}$.

- Determine $d^{-1}(x)$ for $v \geq 0$. What do its input and its output represent?
- Police measure skid marks 72 m long in a 100 km/h zone. How fast was the car going? Was it speeding?
- Solving $\frac{v^2}{200} = 72$ gives $v = \pm 120$. Why does d need the restriction $v \geq 0$ to have an inverse function, and why is $v = -120$ rejected?

A3 Tiered Electricity

An Ontario utility charges \$0.10 per kWh for the first 600 kWh used in a month, and \$0.12 per kWh for every kWh above 600. Let $C(E)$ be the monthly cost, in dollars, for E kWh.

- Write $C(E)$ as a piecewise function. Find the cost of using 900 kWh.
- A monthly bill is \$84. How much electricity was used?
- Plan B charges \$0.10 per kWh if $E \leq 600$, but \$0.12 per kWh on **all** usage if $E > 600$. A customer on plan B says their bill was \$66. Show that this is impossible.

Think It Through (T)

Counterexamples · Error Analysis

T1 Even, Odd, or Neither?

A student claims: "If the only power of x in a function's rule is even, the function is even."

- Show that $p(x) = (x - 3)^2$ is a counterexample. Is p even, odd, or neither?
- Determine whether $q(x) = \frac{x^2 + 1}{x}$ is even, odd, or neither, using $q(-x)$.

- Explain why the only function with domain $\mathbb{R}$ that is both even and odd is $f(x) = 0$.

T2 Spot the Flaw

A student describes how to get $y = -2\sqrt{4 - x} + 1$ from $y = \sqrt{x}$:

"Reflect in the x -axis, stretch vertically by a factor of 2, translate 4 units left, then 1 unit up."

- Apply the student's steps to the point $(9, 3)$ on $y = \sqrt{x}$. Show that the image is not on the new graph.
- Write the equation in the form $y = af(k(x - d)) + c$ and give the correct list of transformations. Find the correct image of $(9, 3)$.
- State the domain and range of $y = -2\sqrt{4 - x} + 1$ in interval notation.

Boss Level

Optional challenge

Nested Absolute Values

Consider the equation $||x - 2| - 3| = k$.

- Solve the equation for $k = 1$.
- Find every value of k for which the equation has exactly three solutions.
- For $0 < k < 3$ the equation has four solutions. Find their sum.



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