

Polynomial Functions Review

Answer key Final answers only. Try first, then check.

CORE SKILLS

- K1** a) As $x \rightarrow -\infty, y \rightarrow \infty$; as $x \rightarrow \infty, y \rightarrow -\infty$; odd
 b) As $x \rightarrow \pm\infty, y \rightarrow \infty$; even
 c) As $x \rightarrow \pm\infty, y \rightarrow -\infty$; neither
-
- K2** a) 3 (third differences are constant: -12)
 b) -2
 c) As $x \rightarrow -\infty, y \rightarrow \infty$; as $x \rightarrow \infty, y \rightarrow -\infty$; at most 2 turning points
-
- K3** a) -2 (order 2), 1 (order 1), $\frac{3}{2}$ (order 1)
 b) Degree 4; leading coefficient -2 ; y -intercept -12
 c) As $x \rightarrow \pm\infty, y \rightarrow -\infty$; crosses at $x = 1$ and $x = \frac{3}{2}$ (touches at $x = -2$)
-
- K4** a) $Q(x) = x^3 - 2x^2 + x + 3$; $R = -13$
 b) $6x^3 + x^2 - 8x + 7 = (2x - 1)(3x^2 + 2x - 3) + 4$
 c) $-\frac{7}{2}$
-
- K5** a) $(3x + 2)(9x^2 - 6x + 4)$
 b) $2x(x - 4)(x^2 + 4x + 16)$
 c) $(x - 5)(x - 3)(x + 3)$

REAL WORLD

- A1** a) $V(x) = x(30 - 2x)(20 - 2x)$; $D = (0, 10)$
 b) $x = 5$ cm or $x = 10 - 5\sqrt{2} \approx 2.9$ cm
 c) $x = 10 + 5\sqrt{2} \approx 17.1$; no: it is outside $(0, 10)$, and both $30 - 2x$ and $20 - 2x$ are negative there.
-
- A2** a) $P(x) = -(x + 2)(x - 1)(x - 5)$
 b) $(-\infty, -2) \cup (1, 5)$
 c) $(-\infty, -2)$: negative sales are impossible. A profit needs more than 1000 and fewer than 5000 bottles.
-
- A3** a) $2x^2 - 5x + 2$ cm² (remainder 0)
 b) $(2x - 1)$ cm and $(x - 2)$ cm; at $x = 5$: 9 cm by 3 cm, height 8 cm
 c) No: a height of 4 cm gives $x = 1$, so the width $x - 2$ is -1 cm. All three dimensions are positive only when $x > 2$, so the height must be greater than 5 cm.

THINK IT THROUGH

- T1** a) $P(1) = -10$: $(x - 1)$ is not a factor; dividing by it leaves remainder -10 .
 b) $P(-1) = 0$ makes $(x + 1)$ a factor, not $(x - 1)$; and the division by $x - 1$ left remainder -10 , which the student dropped.
 c) $P(x) = (x + 1)(x - 2)(x + 4)$; zeros $-4, -1, 2$
-
- T2** a) $y = -\frac{1}{2}(x + 3)(x - 1)^2$
 b) $y = -2(x + 1)^2(x - 3)^2$
 c) $y = -x(x - 2)(x + 2)$, that is $y = -x^3 + 4x$

BOSS LEVEL

- Boss** a) $a = 2, b = 1$: the remainder is $2x + 1$
 b) $P(x) = x^3 - 2x^2 - 3x + 7$
 c) $-2x + 5$

