

Polynomial Functions

Ontario MHF4U • end behaviour, zeros, division, factor theorem, equations and inequalities

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Worksheet, answers &
this cheat sheet



MOST TESTED Factor $P(x) = x^3 - 2x^2 - 5x + 6$ fully and state its zeros. (card 5)

Which factoring? Two cubes **Cubes pattern** Four terms **Grouping** No pattern **Factor theorem**

1 End behaviour and symmetry

USE Even function: $f(-x) = f(x)$,

symmetric in the y -axis. Odd:

$f(-x) = -f(x)$, symmetric about the origin.

Degree and leading coefficient a (left end, right end):

even degree, $a > 0$ **up, up**

even degree, $a < 0$ **down, down**

odd degree, $a > 0$ **down, up**

odd degree, $a < 0$ **up, down**

TRAP An even **degree** does not make an even **function**: $x^4 + x$ is neither.

2 Differences and turning points

USE Degree n : the n th differences are constant and equal $a \cdot n!$. At most $n - 1$ turning points and n zeros.

TRAP Constant 3rd differences of -12 :
 $a = -12/3! = -2$, not -12 .

3 Zeros and their order

USE Odd order: the graph crosses. Even order: it touches and turns back. Leading coefficient: a times the x -coefficients. A family $y = a(\dots)$: one point gives a .

EXAMPLE $y = -(x + 2)^2(2x - 1)$: zeros -2 (touch) and $1/2$ (cross), leading coefficient -2 , y -intercept 4 .

TRAP The factor $(x + 2)$ gives the zero -2 , not 2 .

4 Dividing polynomials

$$P(x) = D(x)Q(x) + R$$

USE Write 0 for each missing power.

Synthetic division by $x - a$ uses a : for $x + 2$ use -2 ; for $2x + 1$ use $-1/2$, then divide the quotient by 2.

TRAP $x^3 - 5x + 2$ has no x^2 term: its coefficients are $1, 0, -5, 2$.

5 Remainder and factor theorems

USE $P(x) \div (x - a)$ leaves remainder

$P(a)$; $x - a$ is a factor exactly when

$P(a) = 0$. Test $\pm$ factors of the constant (over factors of a).

EXAMPLE $P(x) = x^3 - 2x^2 - 5x + 6$:
 $P(1) = 0$, so
 $P = (x - 1)(x^2 - x - 6) = (x - 1)(x - 3)(x + 2)$.

TRAP $P(-1) = 0$ means $x + 1$ is a factor, not $x - 1$.

6 Cubes and grouping

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

USE Common factor first. Four terms: group in pairs.

TRAP $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$: the middle sign is the opposite of the first.

7 Solving equations

USE Factor completely and set each factor to 0. A quadratic factor with $D < 0$ adds no real roots. $ax^4 + bx^2 + c$: let $u = x^2$. In context, keep only roots in the domain.

EXAMPLE
 $x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9) = 0$:
 $x = \pm 2, \pm 3$.

TRAP A box cut from a 20 cm square with side $20 - 2x$ needs $0 < x < 10$: reject roots outside.

8 Inequalities

USE Everything on one side, factor, mark the zeros, test the sign of each interval. At an even-order zero the sign does not change.

EXAMPLE $(x + 1)(x - 2)(x - 5) > 0$:
 $(-1, 2) \cup (5, \infty)$.

TRAP Not factor by factor: $(x - 1)(x - 5) > 0$ is $x < 1$ or $x > 5$, not " $x > 1$ and $x > 5$ ".