

Polynomial Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

End behaviour: even degree, both ends go the same way; odd degree, opposite ways. A positive leading coefficient means the graph rises to the right.

Order of a zero: at a zero of odd order the graph crosses the x -axis; at a zero of even order it touches and turns back.

$(x - 3)(x + 4) > 0$ is **not** solved factor by factor. Mark the zeros on a number line and test the sign of each interval.

Show your work on separate paper.

Core Skills (K)

End Behaviour · Finite Differences · Factored Form · Division · Factoring

K1 End Behaviour & Symmetry

For each function, state the end behaviour, then determine whether the function is even, odd or neither.

- $f(x) = -3x^5$
- $g(x) = 2x^4 - 5x^2 + 1$
- $h(x) = -2x^4 + x$

K2 Finite Differences

The table shows values of a polynomial function f .

x	-1	0	1	2	3	4
$f(x)$	3	-1	1	-3	-25	-77

- Determine the degree of f using finite differences.
- Determine the leading coefficient of f .
- State the end behaviour of f and the maximum number of turning points its graph can have.

K3 Factored Form

Let $f(x) = -(x + 2)^2(x - 1)(2x - 3)$.

- State the zeros of f and the order of each.
- Determine the degree, the leading coefficient and the y -intercept.
- State the end behaviour and the zeros where the graph crosses the x -axis.

K4 Dividing Polynomials

- Use synthetic division to divide $x^4 - 3x^2 + 5x - 7$ by $x + 2$. State the quotient and the remainder.
- Use long division to divide $6x^3 + x^2 - 8x + 7$ by $2x - 1$. Write the result as $P(x) = (2x - 1)Q(x) + R$.

Remainder and factor theorems: dividing $P(x)$ by $x - a$ leaves remainder $P(a)$, and $x - a$ is a factor exactly when $P(a) = 0$. The factor $x + 4$ goes with the zero -4 .

Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

- Without dividing, determine the remainder when $4x^3 - 2x^2 + 3x - 1$ is divided by $2x + 1$.

K5 Factoring Strategies

Factor fully.

- $27x^3 + 8$
- $2x^4 - 128x$
- $x^3 - 5x^2 - 9x + 45$

Real World (A)

Polynomial Equations · Inequalities · Division in Context

A1 Open-Top Box

An open-top box is made from a 30 cm by 20 cm sheet of cardboard by cutting a square of side x cm from each corner and folding up the sides.

- Write the volume $V(x)$ as a product of three factors and state its domain in interval notation.
- Find every cut size that gives a volume of 1000 cm³. Round to the nearest tenth where needed.
- The equation $V(x) = 1000$ has a third solution. Find it. Does it give a box?

A2 Maple Syrup Profit

A maple syrup producer in Ontario models its yearly profit, in thousands of dollars, by $P(x) = -x^3 + 4x^2 + 7x - 10$, where x is the number of bottles sold, in thousands. At high volumes, the extra labour and shipping cost more than the extra bottles earn.

- Factor $P(x)$ fully.
- Solve $P(x) > 0$. Give your answer in interval notation.
- Which part of your answer to (b) has no meaning for the producer? State the numbers of bottles that give a profit.

A3 Storage Box

A rectangular storage box has volume

$V(x) = 2x^3 + x^2 - 13x + 6$, in cubic centimetres, and height $(x + 3)$ cm. Its length and width are also linear expressions in x .

- Divide to find an expression for the area of the base.
- Factor the base area to find the length and width. Then find all three dimensions when $x = 5$.
- A designer wants the box to be 4 cm tall. Is this possible? State every possible height.

Think It Through (T)

Error Analysis · Building Polynomials

T1 Spot the Flaw

A student factors $P(x) = x^3 + 3x^2 - 6x - 8$:

" $P(-1) = -1 + 3 + 6 - 8 = 0$, so $(x - 1)$ is a factor.

Synthetic division by 1 gives $x^2 + 4x - 2$, so

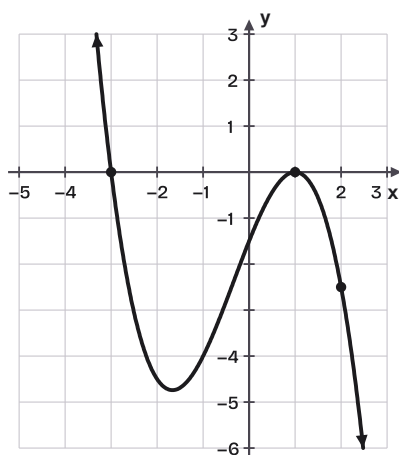
$P(x) = (x - 1)(x^2 + 4x - 2)$."

- Evaluate $P(1)$. What does the result tell you about $(x - 1)$?
- Identify the two errors in the student's work.
- Factor $P(x)$ fully and state its zeros.

T2 Build It Backwards

Determine an equation for each polynomial function.

- The cubic function graphed below, which passes through the marked point $(2, -2.5)$. Each grid square is 1 unit.



- A quartic whose graph touches the x -axis at $x = -1$ and at $x = 3$, and passes through $(1, -32)$.
- An odd function of least degree that has a zero at $x = 2$ and passes through $(1, 3)$.

Boss Level

Optional challenge

Two Remainders

When the polynomial $P(x)$ is divided by $x - 1$, the remainder is 3. When it is divided by $x + 2$, the remainder is -3 .

- When $P(x)$ is divided by $(x - 1)(x + 2)$, the remainder has the form $ax + b$. Find a and b .
- P is a cubic with leading coefficient 1 and $P(0) = 7$. Find $P(x)$ in standard form.
- Find the remainder when this $P(x)$ is divided by $x^2 - 1$.



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