

Rational Functions

Ontario MHF4U • reciprocal functions, asymptotes and holes, equations and inequalities

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Worksheet, answers &
this cheat sheet



MOST TESTED Find the asymptotes, hole and intercepts of $f(x) = \frac{x^2 - x - 2}{x^2 - 4}$. (card 3)

Hole or asymptote?

A factor cancels

Hole

A factor
stays in the
denominator

Vertical asymptote

Numerator
degree one
higher

Oblique asymptote

1 Reciprocal functions $1/f(x)$

USE Zeros of f become vertical asymptotes. Invariant points: $f(x) = \pm 1$. Where f increases, $1/f$ decreases. A vertex (h, k) of f becomes a turning point $(h, 1/k)$.

EXAMPLE $f(x) = x^2 - 4$: $1/f$ has asymptotes $x = \pm 2$ and $y = 0$, and a turning point at $(0, -1/4)$.

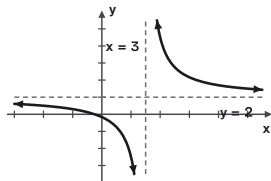
TRAP For $y = 1/f(x)$, invariant points solve $f(x) = \pm 1$; $f(x) = 0$ gives the asymptotes.

TRAP $1/(x^2 + 1)$ has no vertical asymptote: $x^2 + 1$ is never 0.

2 Asymptotes

$$y = \frac{ax + b}{cx + d}; \text{VA } x = -\frac{d}{c}, \text{HA } y = \frac{a}{c}$$

USE Vertical asymptotes: zeros of the denominator that do not cancel. Horizontal: lower numerator degree, $y = 0$; equal, the ratio of leading coefficients; higher, none.



TRAP Equal degrees give HA $y = a/c$: $(2x + 1)/(x - 3)$ has $y = 2$, not $y = 1$.

3 Holes and oblique asymptotes

USE Factor first. A factor that cancels gives a **hole**; take its y from the simplified form. Numerator degree one higher: divide, and the quotient is the oblique asymptote.

EXAMPLE $\frac{x^2 - x - 2}{x^2 - 4} = \frac{x + 1}{x + 2}, x \neq 2$:
hole $(2, 3/4)$, VA $x = -2$, HA $y = 1$.

4 Domain, range and sign

USE Domain: remove asymptotes and holes. Range: also remove a hole's y -value (if not reached elsewhere). Sign chart: zeros of the numerator **and** the denominator.

TRAP A graph may cross its horizontal asymptote; it never crosses a vertical one.

5 Rational equations

USE Multiply **every** term by the LCD. Reject any root that makes an original denominator 0. Work problems: add rates, $1/a + 1/b = 1/t$.

EXAMPLE $x/(x - 2) + 3 = 2/(x - 2)$:
 $x + 3(x - 2) = 2$, so $x = 2$: rejected, no solution.

TRAP Two pipes taking t and $t + 3$ hours: add the rates $1/t + 1/(t + 3)$, never the times.

6 Rational inequalities

USE Get 0 on one side as one fraction. Critical values: zeros of the numerator and of the denominator. Test each interval. Denominator zeros always get round brackets.

EXAMPLE $(x - 3)/(x + 1) \leq 0$: $(-1, 3]$.

TRAP Never multiply both sides by an expression whose sign you don't know: use a sign chart.