

Rational Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Asymptotes of $\frac{ax+b}{cx+d}$: vertical $x = -\frac{d}{c}$, horizontal $y = \frac{a}{c}$. When the numerator's degree is one higher than the denominator's, divide: the quotient is the oblique asymptote.

Reciprocal $y = \frac{1}{f(x)}$: the zeros of f become vertical asymptotes,

and the invariant points are where $f(x) = 1$ or $f(x) = -1$.

Never multiply an inequality by an expression whose sign you don't know. Get 0 on one side and use a sign chart; a zero of the denominator always gets a round bracket.

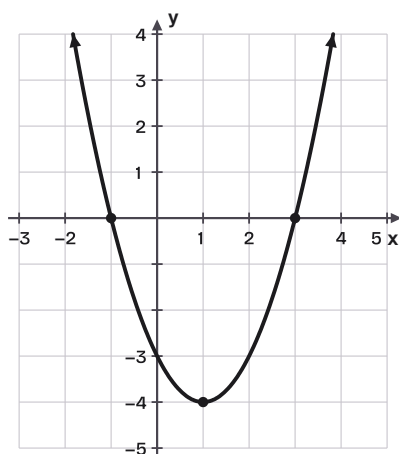
Show your work on separate paper.

Core Skills (K)

Reciprocals · Asymptotes & Holes · Domain & Sign · Equations · Inequalities

K1 Reciprocal of a Quadratic

The graph of $f(x) = x^2 - 2x - 3$ is shown. Each grid square is 1 unit.



- State the equations of the asymptotes of $y = \frac{1}{f(x)}$.
- Determine the coordinates of the invariant points of f and $y = \frac{1}{f(x)}$.
- Determine the coordinates of the local maximum of $y = \frac{1}{f(x)}$.

K2 Asymptotes & Holes

For each function, state the equations of all asymptotes and the coordinates of any hole.

Hole or asymptote? Factor first. A factor that cancels gives a hole, and its y -value comes from the simplified function.

Rational equations: multiply every term by the LCD, then reject any root that makes an original denominator zero.

- $f(x) = \frac{6x-1}{3x+4}$
- $g(x) = \frac{x^2-9}{x^2+x-6}$
- $h(x) = \frac{2x^2+3x-1}{x-1}$

K3 Domain, Range & Sign

- State the domain and range of $f(x) = \frac{2x-1}{x+3}$ in interval notation.
- State the domain and range of $g(x) = \frac{x^2-x-2}{x-2}$ in interval notation.
- Determine the intervals on which $h(x) = \frac{x+2}{(x-1)(x-4)}$ is positive and those on which it is negative.

K4 Rational Equations

Solve each equation. Check every candidate against the restrictions.

- $\frac{3}{x-2} = \frac{5}{x+1}$
- $\frac{x}{x-3} - \frac{x}{x+1} = \frac{12}{(x-3)(x+1)}$
- $\frac{2}{x} + \frac{3}{x+2} = 1$

K5 Rational Inequalities

Solve each inequality. Write the answer in interval notation.

- $\frac{x-5}{x+2} \geq 0$
- $\frac{x+4}{x-1} < 3$

c) $\frac{x^2 - x - 6}{x^2 - 1} \leq 0$

Real World (A)

Work Rates · Average Cost · Medication Levels

A1 Two Pumps

After a spring storm, two pumps drain a flooded underpass in Ottawa. Working together, they drain it in 2 hours. Alone, the older pump takes 3 hours longer than the newer one. Let t be the time, in hours, that the newer pump takes alone.

- Write a rational equation that models the situation.
- Solve the equation. How long does each pump take alone?
- The equation has a second root, $t = -2$. It satisfies the equation and makes no denominator zero. Should it be rejected? Why?

A2 Club Hoodies

A school club orders custom hoodies. The supplier charges a \$600 setup fee plus \$15 per hoodie. For an order of n hoodies, the average cost per hoodie, in dollars, is

$$A(n) = \frac{15n + 600}{n}.$$

n	20	50	100	200
$A(n)$	45	27	21	18

- State the equation of the horizontal asymptote of A . What does it mean for the club?
- How many hoodies must the club order for the average cost to be at most \$20?
- A member says a big enough order will bring the average cost down to \$15. Solve $A(n) = 15$ and respond.

A3 Medication Window

The concentration of a medication in a patient's blood, in mg/L, t hours after a dose is $C(t) = \frac{50t}{t^2 + 4}$, for $t \geq 0$. The medication works while the concentration is at least 10 mg/L.

- Evaluate $C(1)$ and $C(10)$, and state the horizontal asymptote of C . What happens to the concentration in the long run?
- Solve $C(t) \geq 10$. For how many hours does the medication work?
- A doctor wants this dose to reach 15 mg/L. Solve $C(t) = 15$ and interpret the result.

Think It Through (T)

Error Analysis · Building Functions

T1 Spot the Flaw

A student analyses $y = \frac{1}{f(x)}$ for $f(x) = x^2 - 2x + 5$:

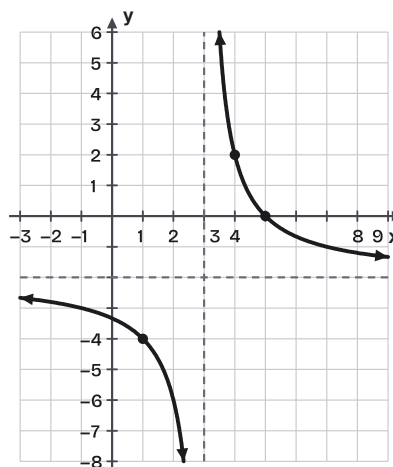
" f is a quadratic, so $\frac{1}{f}$ has two vertical asymptotes. And f increases on $(1, \infty)$, so $\frac{1}{f}$ increases there too."

- Show that f has no zeros. What does this mean for the vertical asymptotes of $y = \frac{1}{f(x)}$?
- Correct the student's second claim.
- State the range of $y = \frac{1}{f(x)}$ and the coordinates of its maximum point.

T2 Build It Backwards

Write the equation of a rational function with each property. In (b) and (c), many answers are possible.

- It has the form $y = \frac{ax + b}{cx + d}$ and the graph below. The asymptotes are dashed. Each grid square is 1 unit.



- A hole at $(1, 4)$ and a vertical asymptote $x = -2$.
- A horizontal asymptote $y = 1$, and a graph that crosses this asymptote. Give the crossing point.

Boss Level

Optional challenge

Exactly One Solution

Consider the equation

$$\frac{x}{x-2} + \frac{k}{x+1} = \frac{3}{(x-2)(x+1)}, \text{ where } k \text{ is a real constant.}$$

- a) Show that every solution satisfies $x^2 + (k+1)x - (2k+3) = 0$, and that $x = 2$ is never a root of this quadratic.
- b) Find the values of k for which the quadratic has a double root.
- c) Find **all** values of k for which the original equation has exactly one solution.

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