

Trigonometric Functions

Ontario MHF4U • radians, graphs, transformations, reciprocal functions, equations, rates of change

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Worksheet, answers &
this cheat sheet



MOST TESTED Solve $2 \sin^2 x - \sin x - 1 = 0$ for $0 \leq x \leq 2\pi$. (card 6)

Which period?

sin, cos, csc, sec

$2\pi/|k|$

tan, cot

$\pi/|k|$

Data or context

max to next max

1 Radians and exact values

$$\pi \text{ rad} = 180^\circ \quad s = r\theta \ (\theta \text{ in radians})$$

USE Find the related acute angle, then take the sign from CAST. $\cos(-x) = \cos x$, $\sin(-x) = -\sin x$. Coterminal: $\theta \pm 2\pi$.

EXAMPLE $5\pi/4$ is in QIII with related angle $\pi/4$: $\sin(5\pi/4) = -\sqrt{2}/2$.

TRAP $\cos(-\pi/4) = +\sqrt{2}/2$: a negative angle does not make every ratio negative.

2 Graphs of sin, cos, tan

USE sin, cos: period 2π , range $[-1, 1]$. tan: period π , zeros at $n\pi$, asymptotes where $\cos x = 0$ ($x = \pi/2 + n\pi$). Cofunction: $\sin(\pi/2 - x) = \cos x$.

TRAP $\tan x$ has period π , and its asymptotes are where $\cos x = 0$, not at its zeros.

3 Transformations

$$y = a \sin[k(x - d)] + c \quad T = \frac{2\pi}{|k|}$$

USE Amplitude $|a|$, midline $y = c$. Factor k out first. For $a > 0$, sine reaches its maximum a quarter period **after** d .

TRAP $\sin(2x - \pi/3) = \sin[2(x - \pi/6)]$: the shift is $\pi/6$, not $\pi/3$.

4 Modelling

USE $a = (\max - \min)/2$, $c = (\max + \min)/2$, $k = 2\pi/T$. With a maximum at $t = d$, use cosine.

EXAMPLE High tide 5 m at 2 h, low 1 m, every 12 h: $h = 2 \cos[\pi/6(t - 2)] + 3$.

TRAP High to low tide is **half** a period: 6 h means $T = 12$ h.

5 Reciprocal functions

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

USE Asymptotes where the denominator is 0. csc, sec: period 2π ; cot: period π . $\csc(kx)$ has asymptotes at $x = n\pi/k$.

TRAP $\csc x$ is $1/\sin x$, not $\sin^{-1} x$.

6 Solving equations

USE Related angle, then every quadrant CAST allows; general solution $+2\pi n$ ($+\pi n$ for tan). Factor quadratic forms; never divide by $\cos x$. Convert csc, sec, cot first.

EXAMPLE $(2 \sin x + 1)(\sin x - 1) = 0$: $x = \pi/2, 7\pi/6, 11\pi/6$.

TRAP $\sin 3x = 1/2$ on $[0, 2\pi]$: $3x$ runs over $[0, 6\pi]$, so there are 6 solutions, not 2.

7 Rates of change

$$\text{average rate} = \frac{f(b) - f(a)}{b - a}$$

USE Instantaneous: shrink the interval. $\sin x$ changes fastest at its zeros and has rate 0 at a max or min.

EXAMPLE $\sin x$ from 0 to $\pi/2$: $(1 - 0)/(\pi/2) = 2/\pi \approx 0.64$.

TRAP A rate of 0 at the top of a Ferris wheel means height isn't changing at that instant, not that the wheel stopped.