

Trigonometric Functions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Radians: π rad = 180° . Arc length $s = r\theta$ only works with θ in radians, and so does your calculator: set it to radian mode.

$y = a \sin(k(x - d)) + c$: amplitude $|a|$, period $\frac{2\pi}{|k|}$, phase shift d ,
midline $y = c$. Factor k out of the bracket before reading d .

A calculator gives one angle. Use CAST to find the second solution in each period, then add multiples of the period for the general solution.

Show your work on separate paper.

Core Skills (K)

Radians · Graphs · Transformations · Reciprocals · Equations

K1 Radians & Exact Values

- Convert 210° to radians and $\frac{7\pi}{4}$ to degrees.
- Evaluate exactly: $\sin \frac{7\pi}{6}$, $\cos \left(-\frac{\pi}{4}\right)$, $\tan \frac{5\pi}{3}$.
- For $\theta = \frac{17\pi}{4}$, determine the principal angle and one negative coterminal angle.

K2 Key Features

- For $y = \tan x$, state the period, the domain and the equations of the vertical asymptotes.
- For $y = \cos x$ on $0 \leq x \leq 2\pi$, state the zeros and the coordinates of every maximum and minimum point.
- Determine the smallest positive value of d such that $\cos x = \sin(x + d)$ for every real x .

K3 Transformations

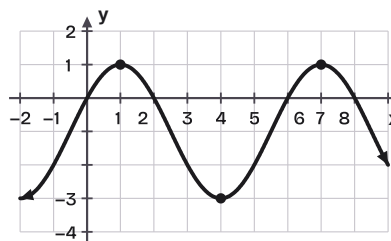
Let $f(x) = -3 \sin \left(2x - \frac{\pi}{3}\right) + 1$.

- Write f in the form $y = a \sin(k(x - d)) + c$. State the amplitude, period, phase shift and the equation of the midline.
- State the range of f and the coordinates of its first maximum point with $x > 0$.

Exact values: find the related acute angle, then take the sign from the quadrant (CAST). $\cos(-x) = \cos x$ but $\sin(-x) = -\sin x$.

Reciprocals: $\csc x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$. Vertical asymptotes sit where the denominator is 0. $\csc x$ is **not** $\sin^{-1} x$.

- Determine an equation of the form $y = a \cos(k(x - d)) + c$ for the graph below. Each grid square is 1 unit.



K4 Reciprocal Functions

- Evaluate exactly: $\csc \frac{5\pi}{4}$, $\sec \frac{2\pi}{3}$, $\cot \frac{\pi}{6}$.
- State the domain, range and period of $y = \sec x$.
- State the period and the equations of the vertical asymptotes of $y = \csc(2x)$.

K5 Trig Equations

- Solve $2 \sin x + \sqrt{3} = 0$ for $0 \leq x \leq 2\pi$.
- Solve $2 \cos^2 x - \cos x - 1 = 0$ for $0 \leq x \leq 2\pi$.
- Determine the general solution of $\csc x = -2$.

Real World (A)

Sinusoidal Models · Rates of Change · Secant in Context

A1 Bay of Fundy Tide

A tide table gives the depth of water D , in metres, at a wharf on the Bay of Fundy, t hours after midnight.

t (h)	0	3	6	9	12	15
D (m)	8	14	8	2	8	14

- Determine an equation of the form $D(t) = a \sin(k(t - d)) + c$ that models the depth.

- b) A fishing boat needs at least 5 m of water to dock. Between midnight and noon, during what times can it **not** dock?
- c) The wharf floods when the depth reaches 15 m. Use your model to find when that first happens.

A2 Fall Fair Ferris Wheel

A Ferris wheel turns once every 8 minutes at a constant speed. A rider boards at the bottom, and their height, in metres, after t minutes is $h(t) = 22 - 20 \cos\left(\frac{\pi}{4}t\right)$.

- a) Determine the average rate of change of height from $t = 0$ to $t = 1$, and from $t = 1$ to $t = 2$. Round to two decimals.
- b) Estimate the instantaneous rate of change of height at $t = 2$, using the interval $2 \leq t \leq 2.001$.
- c) Estimate the instantaneous rate of change at $t = 4$, using $3.999 \leq t \leq 4.001$. A rider concludes that the wheel stopped at $t = 4$. Is that right?

A3 Georgian Bay Lighthouse

A lighthouse on an island in Georgian Bay is 500 m from a straight shoreline. Let θ be the angle between the beam and the line from the lighthouse straight to the shore. When the beam hits the shore, its length is $L(\theta) = 500 \sec \theta$ metres.

- a) Evaluate $L\left(\frac{\pi}{3}\right)$ and $L\left(\frac{\pi}{4}\right)$. Give the second to the nearest tenth of a metre.
- b) The beam turns once every 20 s, so $\theta = \frac{\pi}{10}t$ after t seconds. For $0 \leq t < 5$, when is the beam 1000 m long?
- c) Evaluate L at $t = 5$ s. What happens to the lit spot on the shore at that moment, and for $5 < t < 15$?

Think It Through (T)

Error Analysis · Building Functions

T1 Spot the Flaw

A student solves $2 \sin x \cos x = \cos x$ for $0 \leq x \leq 2\pi$:
"Divide both sides by $\cos x$: $2 \sin x = 1$, so $\sin x = \frac{1}{2}$. The solutions are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$."

- a) Show that $x = \frac{\pi}{2}$ satisfies the equation.
- b) Identify the student's error, then solve the equation correctly.
- c) Without solving again, how many solutions does the equation have for $0 \leq x \leq 4\pi$? Explain.

T2 Build It Backwards

Write an equation for each function. Many answers are possible.

- a) A sine function with maximum value 7, minimum value -1 , period π , and a maximum at $x = \frac{\pi}{3}$.

- b) The same graph as (a), written as a cosine function.
- c) A trig function with period $\frac{\pi}{2}$, vertical asymptotes exactly at $x = \frac{n\pi}{2}$ ($n \in \mathbb{Z}$), whose graph passes through $\left(\frac{\pi}{8}, 1\right)$.

Boss Level

Optional challenge

Squeezed Solutions

Let k be a positive integer. Consider the equation $\sin(kx) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$.

- a) For $k = 3$, list every solution.
- b) For $k = 3$, find the sum of all the solutions.
- c) For a general positive integer k , find the number of solutions and their sum, in terms of k .



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