

Trigonometric Identities

Ontario MHF4U • reciprocal, Pythagorean, compound and double angle identities, proofs and equations

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Worksheet, answers &
this cheat sheet



MOST TESTED Given $\sin A = 3/5$ with A in Quadrant II, find $\sin 2A$ and $\cos 2A$. (card 4)

Which $\cos 2A$?

Only sin elsewhere

$$1 - 2\sin^2 A$$

Only cos elsewhere

$$2\cos^2 A - 1$$

Both, or a
difference of
squares

$$\cos^2 A - \sin^2 A$$

1 Reciprocal, quotient, cofunction

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

$$\sin(\pi/2 - x) = \cos x$$

$$\tan(\pi/2 - x) = \cot x$$

TRAP sec goes with cos, csc with sin:
 $\sec x / \csc x = \sin x / \cos x = \tan x$.

2 Pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x$$

USE A square root gives $\pm$: the quadrant picks the sign.

EXAMPLE $\cos \theta = -7/25$ in QIII:
 $\sin \theta = -\sqrt{1 - 49/625} = -24/25$.

TRAP $\sqrt{1 - \cos^2 \theta} = |\sin \theta|$, not $1 - \cos \theta$.

3 Compound angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

EXAMPLE
 $\cos 75^\circ = \cos(45^\circ + 30^\circ) = (\sqrt{6} - \sqrt{2})/4$.

TRAP $\cos(A - B) \neq \cos A - \cos B$, and its formula has a **plus**:
 $\cos A \cos B + \sin A \sin B$.

4 Double angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$$

EXAMPLE $\sin A = 3/5$ in QII:
 $\cos A = -4/5$, $\sin 2A = -24/25$,
 $\cos 2A = 7/25$.

TRAP $\sin 2A \neq 2 \sin A$: for $\sin A = 2/3$,
 $2 \sin A = 4/3$, which no sine can equal.

5 Proving identities

$$\frac{\sin 2x}{1 + \cos 2x} = \frac{2 \sin x \cos x}{2 \cos^2 x} = \tan x$$

USE Work on **one side at a time**. Rewrite in sin and cos, combine fractions, factor, use a Pythagorean swap.

TRAP Squaring both sides or moving terms across assumes what you are proving: that is not a proof.

6 Solving with identities

USE Reduce to one function and one angle, factor, then solve each factor. Never divide by $\cos x$: factor it out.

EXAMPLE $\sin 2x = \cos x$:
 $\cos x(2 \sin x - 1) = 0$, so
 $x = \pi/6, \pi/2, 5\pi/6, 3\pi/2$.

TRAP Dividing $\sin 2x = \cos x$ by $\cos x$ loses $x = \pi/2$ and $3\pi/2$.