

Trigonometric Identities Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 50 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Reciprocal and quotient: $\csc x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$,
 $\cot x = \frac{\cos x}{\sin x}$, $\tan x = \frac{\sin x}{\cos x}$. Cofunction: $\sin\left(\frac{\pi}{2} - x\right) = \cos x$.

Compound angle: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$,
 $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$,
 $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$.

Proofs: work on one side at a time, never on the equation you are proving. And $\cos(A - B) \neq \cos A - \cos B$, $\sin 2A \neq 2 \sin A$.

Show your work on separate paper.

Core Skills (K)

Reciprocal & Quotient · Pythagorean · Compound Angle · Double Angle · Equations

K1 Reciprocal, Quotient & Cofunction

Simplify each expression to a single trig ratio or a constant.

- $\frac{\sec x}{\csc x}$
- $\cot x \sec x \sin x$
- $\tan\left(\frac{\pi}{2} - x\right) \sin x$

K2 Pythagorean Identity

For (a) and (b), $\cos \theta = -\frac{7}{25}$ and $\pi < \theta < \frac{3\pi}{2}$.

- Determine $\sin \theta$ exactly.
- Determine $\tan \theta$ and $\csc \theta$ exactly.
- Simplify $\frac{\sec^2 x - 1}{\sec^2 x}$ to a single term.

K3 Compound Angles

Angles A and B have the exact values in the table.

angle	A	B
$\sin$	$\frac{8}{17}$	$\frac{3}{5}$
$\cos$	$\frac{15}{17}$	$-\frac{4}{5}$

- Determine $\sin(A + B)$ exactly.
- Determine $\cos(A - B)$ exactly.
- Evaluate $\tan \frac{5\pi}{12}$ exactly.

K4 Double Angles

For (a) and (b), $\sin A = \frac{2}{3}$ and $\frac{\pi}{2} < A < \pi$.

Pythagorean: $\sin^2 x + \cos^2 x = 1$ and $1 + \tan^2 x = \sec^2 x$. A square root gives $\pm$; the quadrant picks the sign.

Double angle: $\sin 2A = 2 \sin A \cos A$,
 $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$,
 $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$.

- Determine $\sin 2A$ exactly.
- Determine $\cos 2A$ and $\tan 2A$ exactly.
- Write each as a single trig ratio, then evaluate exactly:
 $2 \cos^2 \frac{\pi}{8} - 1$ and $\frac{2 \tan 15^\circ}{1 - \tan^2 15^\circ}$.

K5 Equations with Identities

Solve for $0 \leq x \leq 2\pi$. Give exact values.

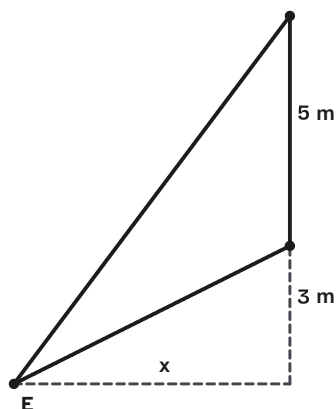
- $2 \sin^2 x + 3 \cos x - 3 = 0$
- $\sin 2x = \cos x$
- $\cos 2x = 3 \sin x - 1$

Real World (A)

Compound Angles · Double Angles · Equations in Context

A1 Mural View

A mural on a Toronto building starts 3 m above your eye level and ends 8 m above it. You stand x metres from the wall. The angles of elevation to the bottom and top of the mural are α and β , so the mural fills an angle $\theta = \beta - \alpha$ of your view.



- Show that $\tan \theta = \frac{5x}{x^2 + 24}$.
- Find the two distances where $\tan \theta = \frac{1}{2}$, and the angle θ to the nearest tenth of a degree.
- A photographer wants the mural to fill a 45° angle of view. How far from the wall should they stand?

A2 Ladder Check

A 6 m ladder leans against a vertical wall and makes an angle θ with the level ground, $0^\circ < \theta < 90^\circ$. The height of its top plus the distance of its foot from the wall is 8 m.

- Show that $\sin 2\theta = \frac{7}{9}$.
- Find both possible angles θ to the nearest tenth of a degree, and the height of the top of the ladder for each, to the nearest centimetre.
- A safety guideline says a ladder should make an angle between 70° and 80° with the ground. Can this ladder meet the 8 m condition safely? Find the exact value of height plus distance at 75° .

A3 Indoor Kick

A soccer player kicks a ball from the ground at 20 m/s, at an angle θ above the horizontal. Ignoring air resistance, it lands $d(\theta) = \frac{800 \sin \theta \cos \theta}{9.8}$ metres away on level ground.

- Write $d(\theta)$ using a single sine function. Find the angle that gives the longest kick, and that distance to the nearest tenth of a metre.

- Find both angles that land the ball 30 m away, to the nearest tenth of a degree.
- The game is in an indoor fieldhouse with a 15 m roof. The ball's maximum height is $h = \frac{400 \sin^2 \theta}{19.6}$ metres. Which angle from (b) can the player use?

Think It Through (T)

Error Analysis · Building Identities

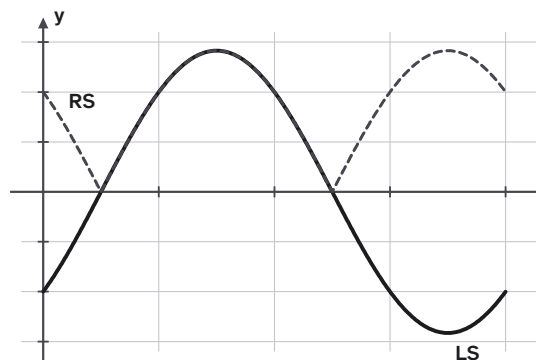
T1 Spot the Flaw

A student claims that $\sin x - \cos x = \sqrt{1 - \sin 2x}$ is an identity:

"Square both sides:

$\sin^2 x - 2 \sin x \cos x + \cos^2 x = 1 - \sin 2x$. The left side equals $1 - \sin 2x$, the same as the right side. Therefore the identity is true."

The graph shows $y = \sin x - \cos x$ (LS) and $y = \sqrt{1 - \sin 2x}$ (RS) for $0 \leq x \leq 2\pi$. Vertical grid lines are $\frac{\pi}{2}$ apart and horizontal grid lines are 0.5 apart.



- Evaluate both sides at $x = 0$. What does this show about the claim?
- State the interval in $[0, 2\pi]$ on which the two sides are equal. Give exact endpoints.
- Identify the flaw in the student's method. Then write the identity that the squaring step does prove.

T2 Build It Backwards

Find the constants. Each identity must hold for every real number x .

- Find a and b such that $\sin^2 x = a + b \cos 2x$.
- Find $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ such that $\sin x + \sqrt{3} \cos x = R \sin(x + \alpha)$.
- A student says the largest value of $\sin x + \sqrt{3} \cos x$ is $1 + \sqrt{3}$, since $\sin x \leq 1$ and $\sqrt{3} \cos x \leq \sqrt{3}$. Use (b) to explain why the student is wrong. Find the largest value and the smallest positive x where it occurs.

Boss Level

Optional challenge

Telescoping CosinesLet $P = \cos 20^\circ \cos 40^\circ \cos 80^\circ$.

- a) Show that $8 \sin 20^\circ \cdot P = \sin 160^\circ$.
- b) Find the exact value of P .
- c) Find the exact value of $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7}$.

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