

# Acute Triangle Trigonometry

Ontario MPM2D • Trigonometry: the sine law and the cosine law in acute triangles

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Worksheet, answers &  
this cheat sheet



**MOST TESTED** In acute  $\triangle ABC$ ,  $b = 9$ ,  $c = 10$  and  $\angle A = 50^\circ$ . Find  $a$  and  $\angle B$ . (card 4)

Which law?

A side and its **opposite** angle

**Sine law**

Two sides and the angle **between**

**Cosine law**

Three sides

**Cosine law**

## 1 Sine law: a side

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**USE** AAS: a side with its **opposite** angle.

ASA: find the third angle first, so the known side has a pair.

**EXAMPLE**  $\angle A = 50^\circ$ ,  $\angle B = 65^\circ$  and  $a = 10$  give  
 $b = 10 \sin 65^\circ / \sin 50^\circ \approx 11.8$ .

**TRAP** Each side pairs with the angle **across** from it, never an angle at one of its ends.

**TRAP** Sides are proportional to the **sines**:  $a/A = b/B$  is false.

## 2 Sine law: an angle

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

**USE** Put the sines on top, then use  $\sin^{-1}$ . Keep full calculator values; round at the end.

**EXAMPLE**  $a = 13$ ,  $b = 10$ ,  $\angle A = 72^\circ$ :  
 $\sin B = 10 \sin 72^\circ / 13$ , so  $\angle B \approx 47.0^\circ$ .

**TRAP**  $\sin B = b \sin A / a$ , not  $a \sin A / b$ : that can exceed 1 and give a calculator error.

## 3 Cosine law

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

**USE** SAS: the third side. SSS: an angle.  $c$  is the side **opposite**  $C$ . At  $C = 90^\circ$  it becomes  $c^2 = a^2 + b^2$ .

**EXAMPLE** Sides 6 and 8,  $50^\circ$  between:  
 $c^2 = 100 - 96 \cos 50^\circ \approx 38.3$ , so  $c \approx 6.2$ .

**TRAP** Sides 6, 8,  $50^\circ$  between:  
 $c^2 = 100 - 96 \cos 50^\circ$ , not  
 $(100 - 96) \cos 50^\circ$ . Multiply first.

## 4 Using both laws

**USE** Name the case first. SAS: cosine law for the third side, then the sine law for an angle. Angles add to  $180^\circ$ .

**EXAMPLE**  $b = 9$ ,  $c = 10$ ,  $\angle A = 50^\circ$ :  
 $a^2 = 81 - 180 \cos 50^\circ$ ,  $a \approx 8.1$ ;  
 $\sin B = 9 \sin 50^\circ / a$ ,  $\angle B \approx 58.6^\circ$ .

**TRAP** SAS has no side with its opposite angle, so the sine law can't start: use the cosine law first.

## 5 Bearings

**USE** Measured clockwise from **north**, with three digits ( $030^\circ$ ). Draw a north line at every point; the north lines are parallel.

**EXAMPLE**  $030^\circ$ , then  $160^\circ$ : the angle inside the triangle at the turn is  
 $180^\circ - (160^\circ - 30^\circ) = 50^\circ$ .

**TRAP**  $030^\circ$  then  $160^\circ$ : the turn angle is  $50^\circ$ , not the difference,  $130^\circ$ .

## 6 Area and chained triangles

$$\text{Area} = \frac{1}{2}ab \sin C$$

**USE**  $C$  is the angle **between**  $a$  and  $b$ . Two triangles: solve one, then carry the shared side into the next.

**EXAMPLE** Sides 12 and 15,  $40^\circ$  between:  
 $\frac{1}{2}(12)(15) \sin 40^\circ \approx 57.9$ .

**TRAP** Use  $\sin C$ :  $\frac{1}{2}ab \cos C$  and  $\frac{1}{2}ab$  are wrong unless  $C = 90^\circ$  (then  $\frac{1}{2}ab$  works).