

Acute Triangle Trigonometry Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Sine law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Each side is paired with the angle

opposite it. To find an angle, put the sines on top: $\frac{\sin B}{b} = \frac{\sin A}{a}$.

Which law? A side and its opposite angle are both known (AAS, ASA, SSA): sine law. SAS or SSS: cosine law.

Most common trap: in the cosine law, multiply $2ab \cos C$ **before** subtracting.

Use degree mode, keep full calculator values until the end, then round lengths to one decimal place and angles to the nearest tenth of a degree.

Show your work on separate paper.

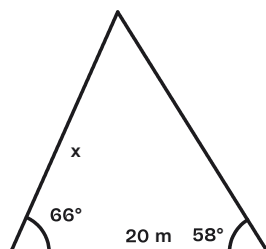
Core Skills (K)

Sine Law · Cosine Law · Choosing the Law

K1 Sine Law: Find the Side

Round lengths to one decimal place.

- In $\triangle ABC$, $\angle A = 52^\circ$, $\angle B = 71^\circ$ and $a = 12$ cm. Find b .
- In $\triangle PQR$, $\angle P = 48^\circ$, $\angle Q = 63^\circ$ and $PQ = 15$ cm. Find QR .
- Find x .



K2 Sine Law: Find the Angle

In acute $\triangle ABC$, $a = 11$ cm, $b = 9$ cm and $\angle A = 64^\circ$.

- Find $\angle B$.
- Find $\angle C$.
- Find c .

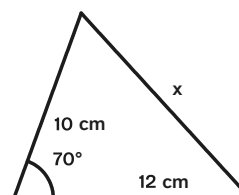
K3 Cosine Law: Find the Side

- In $\triangle DEF$, $d = 8$ cm, $e = 11$ cm and $\angle F = 47^\circ$. Find f .

Cosine law: $c^2 = a^2 + b^2 - 2ab \cos C$, where C is the angle between a and b . For an angle: $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$.

Area = $\frac{1}{2}ab \sin C$. **Bearings** are measured clockwise from north and written with three digits (040°).

- Find x .



- In $\triangle XYZ$, $x = 9$, $y = 12$ and $\angle Z = 90^\circ$. Use the cosine law to find z .

K4 Cosine Law: Find the Angle

In $\triangle ABC$, $a = 7$ cm, $b = 8$ cm and $c = 10$ cm. Round angles to the nearest tenth of a degree.

- Find $\angle C$.
- Find $\angle A$.
- Another triangle has sides $a = 5$, $b = 6$ and $c = 7$. Determine $\cos C$ as a fraction in lowest terms.

K5 Which Law First?

The table gives the known measures of four acute triangles ABC . Lengths are in centimetres.

Triangle 1	$\angle A = 40^\circ$	$\angle B = 75^\circ$	$a = 10$
Triangle 2	$a = 6$	$b = 9$	$\angle C = 58^\circ$
Triangle 3	$a = 6$	$b = 7$	$c = 8$
Triangle 4	$a = 12$	$b = 10$	$\angle A = 70^\circ$

- For each triangle, state the case (AAS, SAS, SSS or SSA) and the law you would use first.
- Triangle 2: find c .
- Triangle 4: find $\angle B$.

Real World (A)

Bearings · Area · Multi-Step Problems

A1 Back to Tobermory

A sailboat leaves Tobermory harbour (T) and sails 14 km on a bearing of 040° to a buoy P . It then turns and sails 9 km on a bearing of 150° to point Q .

- Sketch the route and find $\angle TPQ$, the angle of the triangle at P .
- How far is the boat from the harbour, and on what bearing must it sail to return directly? Round the bearing to the nearest degree.
- Another boat's log reads: "14 km on one bearing, then 9 km on another; finished 25 km from the harbour." What can you conclude about the log?

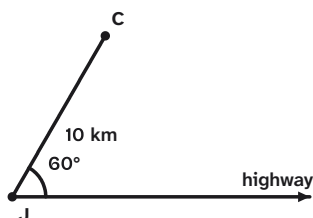
A2 Community Garden

A community garden in Mississauga is a triangle. Two of its sides run along paths and measure 18 m and 25 m, with an angle of 62° between them.

- Find the area of the garden.
- The third side needs a fence that costs \$14.50 per metre. Find the length of that side and the cost of the fence.
- The city wants a 250 m^2 garden along the same two paths, with sides of 18 m and 25 m, changing only the angle between them. Is this possible? Explain.

A3 Signal on the Highway

A straight highway starts at Junction J and runs east. A cell tower C stands 10 km from J , and segment JC makes a 60° angle with the highway. The tower's signal reaches 12 km.



- Find the shortest distance from the tower to the highway.
- A driver heading east loses the signal x km from J . Use the cosine law to write an equation in x , then solve it.
- Which solution answers the question, and what would the other one mean? State the stretch of highway, measured from J , where a driver has signal.

Think It Through (T)

Error Analysis · Counterexample · Build a Triangle

T1 Spot the Flaw

In $\triangle ABC$, $a = 7$ cm, $b = 9$ cm and $\angle C = 52^\circ$. A student writes:

$$c^2 = 7^2 + 9^2 - 2(7)(9) \cos 52^\circ = 130 - 126 \cos 52^\circ = 4 \cos 52^\circ \approx 2.46, \text{ so } c \approx 1.6 \text{ cm.}$$

- Without redoing the calculation, explain how you can tell that 1.6 cm is impossible.

- Identify the error and find c .
- A second student says: "The cosine law is Pythagoras with an extra term, so c should be about $\sqrt{7^2 + 9^2} \approx 11.4$ cm." For which angle C is $c^2 = a^2 + b^2$ true? Explain why $c < 11.4$ cm here.

T2 Angles and Opposite Sides

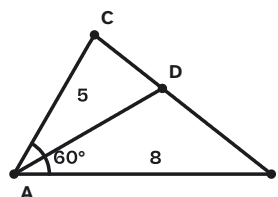
- A student claims: "If one angle of a triangle is twice another, the side opposite it is twice as long." Give a counterexample.
- In $\triangle ABC$, $a = 6$ cm and $\angle A = 40^\circ$. Choose a length b for which no triangle exists, and explain what goes wrong in the sine law.
- Build an acute triangle with whole-number sides, not all equal, that has a 60° angle. Use the cosine law to show it works.

Boss Level

Optional challenge

Bisect the Angle

In $\triangle ABC$, $AB = 8$, $AC = 5$ and $\angle A = 60^\circ$. The bisector of $\angle A$ meets BC at D .



- Find BC and the exact area of $\triangle ABC$.
- Find the exact length of AD .
- Find BD and DC . Is D the midpoint of BC ?



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