

Analytic Geometry

Ontario MPM2D • Analytic Geometry: midpoint, length, circles and properties of figures

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Worksheet, answers &
this cheat sheet



MOST TESTED Find the shortest distance from $P(7, 0)$ to the line $y = 2x + 1$. (card 7)

Which tool?

Is there a right angle?

Product of slopes

Equal sides, perimeter, on a circle?

Length formula

A point halfway, or a bisector

Midpoint formula

1 Midpoint and length

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE $A(-1, 2)$, $B(5, 10)$: $M(2, 6)$,
 $AB = \sqrt{36 + 64} = 10$.

TRAP The midpoint **averages**:
 $(-1 + 5)/2 = 2$, not $(5 - (-1))/2 = 3$.

TRAP Midpoint $M(2, 6)$ and endpoint $A(-1, 2)$:
 $B = (2 \cdot 2 + 1, 2 \cdot 6 - 2) = (5, 10)$.
Averaging A and M gives a quarter point.

2 Circle at the origin

$$x^2 + y^2 = r^2$$

USE A point on it gives r^2 . Intercepts $(\pm r, 0)$, $(0, \pm r)$. Inside: $x^2 + y^2 < r^2$; outside: $> r^2$.

EXAMPLE Through $(8, -6)$:
 $r^2 = 64 + 36 = 100$, so $x^2 + y^2 = 100$
and $r = 10$.

TIP The perpendicular bisector of any chord passes through the centre.

TRAP $x^2 + y^2 = 20$ has $r = 2\sqrt{5}$, not 20.

TRAP Diameter from $(-6, 8)$ to $(6, -8)$:
 $r = 10$, so $x^2 + y^2 = 100$, not $20^2 = 400$.

3 Slopes and triangles

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

parallel: $m_1 = m_2$

perpendicular: $m_1 m_2 = -1$

USE Sides by lengths: scalene, isosceles, equilateral. Angles by slopes: a right angle needs $m_1 m_2 = -1$.

EXAMPLE $A(0, 0)$, $B(4, 2)$, $C(3, 4)$:
 $m_{AB} = 1/2$, $m_{BC} = -2$, product -1 :
right angle at B .

TRAP $AB = BC$ makes it isosceles, not right-angled. Test the slopes; never trust the picture.

4 Quadrilaterals

one pair of parallel sides **trapezoid**

both pairs parallel **parallelogram**

parallelogram, a right angle **rectangle**

four equal sides **rhombus**

rhombus, a right angle **square**

TRAP Four equal sides make a rhombus. It is a square only if adjacent slopes multiply to -1 .

5 Lines in a triangle

median: vertex to the midpoint of the opposite side **meet at the centroid**

altitude: from a vertex, $\perp$ to the opposite side **orthocentre**

perpendicular bisector: through a side's midpoint, $\perp$ to it **circumcentre**

TRAP Opposite side slope $3/4$: the altitude's slope is $-4/3$, not $3/4$.

6 Proving properties

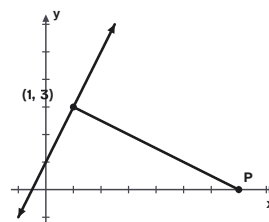
USE Midsegment: parallel to the third side and half its length. The circumcentre is the same distance from all three vertices. A line from the centre $\perp$ to a chord bisects it.

TRAP One numerical triangle only **verifies**. A proof uses general vertices such as $(0, 0)$, $(2a, 0)$, $(2b, 2c)$.

TRAP The point equidistant from the vertices is the circumcentre, not the centroid. It can lie outside.

7 Distance to a line

USE Perpendicular through P (negative reciprocal slope); find where it meets the line; measure from P to that point.



EXAMPLE $P(7, 0)$, $y = 2x + 1$:
perpendicular $y = -x/2 + 3.5$ meets it at $(1, 3)$; distance $\sqrt{36 + 9} = 3\sqrt{5} \approx 6.7$.

TRAP Not vertically: from $(7, 0)$ straight up to $y = 2x + 1$ is 15, but the shortest distance is $3\sqrt{5}$.

TRAP On a **segment**, check the foot lies on it; if not, use the nearest endpoint.