

Factoring

Ontario MPM2D • Quadratic Relations: common factors, trinomials, special products and grouping

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Worksheet, answers &
this cheat sheet



MOST TESTED Factor completely: $6x^2 + 3x - 18$. (card 4)

After the GCF

Two squares,
subtracted

$$(a - b)(a + b)$$

Three terms, x^2
alone

Product and sum

Three terms, ax^2
with $a \neq 1$

Decomposition

1 Common factor (GCF)

USE Take out the largest number and the lowest power of each shared variable. Divide every term by it.

EXAMPLE

$$12x^3y - 18x^2y^2 + 6x^2y = 6x^2y(2x - 3y + 1).$$

TRAP $5a^2 + 5a = 5a(a + 1)$: the term equal to the GCF leaves 1, not nothing.

TRAP The GCF stays in the answer: $4x^2 - 12x = 4x(x - 3)$, never $(x - 3)$.

2 Grouping

USE Four terms: group in pairs, take the GCF of each pair, then take out the common bracket. If the brackets don't match, try another pairing.

EXAMPLE

$$x^3 + 5x^2 - 2x - 10 = x^2(x + 5) - 2(x + 5) = (x + 5)(x^2 - 2).$$

TRAP Taking out a negative flips **both** signs: $-3x + 12 = -3(x - 4)$, not $-3(x + 12)$.

3 $x^2 + bx + c$

$$x^2 + bx + c = (x + m)(x + n) \\ mn = c \quad m + n = b$$

$c > 0, b > 0$ **both +**

$c > 0, b < 0$ **both -**

$c < 0$ **opposite signs**

EXAMPLE $x^2 - 3x - 28$: $(-7)(4) = -28$ and $-7 + 4 = -3$, so $(x - 7)(x + 4)$.

TIP When $c < 0$, the number with the larger size has the sign of b .

TRAP $x^2 - 4x - 21 = (x - 7)(x + 3)$, not $(x - 7)(x - 3)$: $c < 0$ needs opposite signs.

4 $ax^2 + bx + c$: decomposition

USE GCF first. Find m, n with $mn = ac$ and $m + n = b$. Split bx into $mx + nx$, then group.

EXAMPLE $2x^2 + 7x + 3$: $ac = 6$ and

$6 + 1 = 7$, so

$$2x^2 + 6x + x + 3 = (2x + 1)(x + 3).$$

TRAP 6 and 1 split the middle term. They don't go into brackets: $(x + 6)(x + 1)$ is wrong.

TRAP $3x(x + 2) - 4x - 8$: take out -4 , giving $-4(x + 2)$, so both brackets match.

5 Special products

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^2 \pm 2ab + b^2 = (a \pm b)^2$$

USE Square-root the coefficients: $25x^2 - 16 = (5x - 4)(5x + 4)$. A sum of squares $a^2 + b^2$ does not factor.

TRAP $4x^2 + 13x + 9$ is not $(2x + 3)^2$: the middle term would be $12x$. Check $2ab$ first.

6 Factor completely

USE 1. GCF first (negative if the first term is). 2. Count terms: two, difference of squares; three, trinomial; four, grouping. 3. Try to factor every bracket again.

EXAMPLE

$$3x^3 - 12x = 3x(x^2 - 4) = 3x(x - 2)(x + 2).$$

TIP Expand your answer to check it.

TRAP $x^4 - 81 = (x^2 + 9)(x^2 - 9)$ is not finished: $(x^2 + 9)(x - 3)(x + 3)$.

TRAP Taking out -2 flips every sign: $-2x^2 + 6x + 8 = -2(x^2 - 3x - 4)$.