

Factoring Algebraic Expressions Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

GCF first. Every factor you remove stays in the answer, and a term equal to the GCF leaves a 1: $8y^2 + 4y = 4y(2y + 1)$.

Special cases: $a^2 - b^2 = (a - b)(a + b)$ and $a^2 \pm 2ab + b^2 = (a \pm b)^2$. Check that the middle term is $2ab$ before using the square pattern.

$(a + b)^2$ is **not** $a^2 + b^2$, and a sum of squares $a^2 + b^2$ does not factor over the integers.

Show your work on separate paper.

Core Skills (K)

GCF & Grouping · Simple Trinomials · Decomposition · Special Cases · Factoring Completely

K1 Common Factors & Grouping

Factor.

- a) $14a^3b^2 - 21a^2b^4 + 7a^2b^2$
- b) $x^3 - 4x^2 - 3x + 12$
- c) $6xy + 15 - 10y - 9x$

K2 Simple Trinomials

Factor.

- a) $x^2 + 11x + 24$
- b) $x^2 - 2x - 35$
- c) $x^2 - 13xy + 36y^2$

K3 Complex Trinomials

Factor completely.

- a) $3x^2 + 10x + 8$
- b) $6x^2 - 7x - 20$
- c) $20x^2 - 26x + 6$

K4 Special Cases?

Factor. Not every expression is a special case.

- a) $49x^2 - 36y^2$
- b) $4x^2 - 20x + 25$
- c) $9x^2 + 30x + 16$

K5 Factor Completely

Factor completely. Each expression needs more than one step.

- a) $-2x^2 + 14x - 20$
- b) $16x^4 - 1$

$x^2 + bx + c$: two numbers with product c and sum b . $ax^2 + bx + c$: two numbers with product ac and sum b ; use them to split the middle term, then group.

Factor completely: after each step, test every bracket again. Expand your answer to check it.

c) $x^3 + 2x^2 - 9x - 18$

Real World (A)

Revenue · Area · Volume

A1 Bike Rentals

A bike-rental shop in Toronto models its daily revenue as $R = -6x^2 - 10x + 1200$ dollars, where x is the number of \$2 increases to its daily price of \$30 per bike.

- a) Factor R completely.
- b) Rewrite R as a product of two factors, one of which is the price per bike, $30 + 2x$. What does the other factor represent? How many bikes are rented at \$30?
- c) The factored form gives $R = 0$ at $x = \frac{40}{3}$. Can the shop set this price? Find the largest number of increases for which at least one bike is rented, and the revenue then.

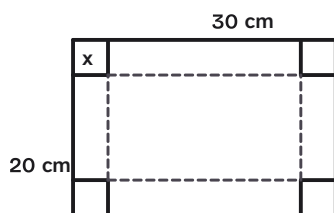
A2 Plaza Paving

A square plaza in Kingston has side length s metres. A square fountain of side f metres sits in the middle, and the rest of the plaza is paved.

- a) Write the paved area in factored form.
- b) The plaza is 46 m wide and the fountain is 14 m wide. Paving costs \$85 per m^2 . Use your factored form to find the paving cost.
- c) A designer sketches a fountain 50 m wide for the same plaza. Evaluate your factored form and say what the result tells the designer.

A3 Cardboard Tray

An open-top tray is made from a 30 cm by 20 cm sheet of cardboard: a square of side x cm is cut from each corner, then the sides are folded up. The tray's volume is $V = 4x^3 - 100x^2 + 600x \text{ cm}^3$.



- Factor V completely.
- Rewrite your answer as height $\times$ width $\times$ length of the tray. Then find the volume when $x = 4$.
- The factored form also gives $V = 0$ at $x = 15$. Can 15 cm squares be cut from this sheet? State the realistic values of x .

Think It Through (T)

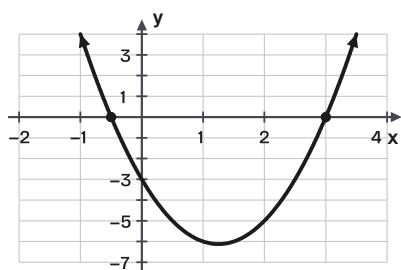
Error Analysis · Counterexamples

T1 Spot the Flaw

A student factors $2x^2 - 5x - 3$:

" $ac = -6$. The numbers -6 and 1 multiply to -6 and add to -5 , so $2x^2 - 5x - 3 = (x - 6)(x + 1)$."

The graph of $y = 2x^2 - 5x - 3$ is shown. Each grid square is 1 unit.



- Use the graph to show that the student's answer is wrong.
- Identify the student's error, then factor $2x^2 - 5x - 3$ correctly.
- The student's shortcut does work for $x^2 - 5x - 6$. Explain why it works there but not for $2x^2 - 5x - 3$.

T2 Test the Claim

A student claims $(x + 3)^2 = x^2 + 9$ and tests it with a table.

x	0	1	-3
$(x + 3)^2$	9		
$x^2 + 9$	9		

- The student tested only $x = 0$. Complete the table. Does it support the claim?
- Expand $(x + 3)^2$. For which values of x are the two expressions equal?
- A second student claims $x^2 + 9 = (x + 3)(x - 3)$. Give a counterexample, then explain why $x^2 + 9$ cannot be factored over the integers.

Boss Level

Optional challenge

Differences of Squares

Some whole numbers can be written as a difference of two perfect squares, for example $40 = 7^2 - 3^2$.

- Find positive integers m and n with $m^2 - n^2 = 96$ and $m - n = 2$.
- Find all pairs of positive integers (m, n) with $m^2 - n^2 = 96$.
- Explain why no integers m and n satisfy $m^2 - n^2 = 98$.



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