

Linear Systems

Ontario MPM2D • Analytic Geometry: solving linear systems by graphing, substitution and elimination

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Worksheet, answers &
this cheat sheet



MOST TESTED A school sold 50 tickets for \$480. Adult tickets cost \$12 and student tickets \$8. How many of each were sold? (card 6)

Which method?

A variable is alone
or has coefficient
 ± 1

Substitution

Coefficients match
or are small
multiples

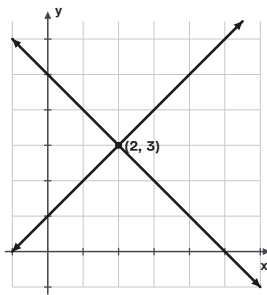
Elimination

You need only
how many
solutions

Compare slopes

1 Solution of a system

USE A point that satisfies **both** equations: the **point of intersection** of the lines. To graph, write each line as $y = mx + b$.



EXAMPLE $y = x + 1$ and $y = -x + 5$ cross at $(2, 3)$, since $3 = 2 + 1$ and $3 = -2 + 5$

TRAP $(1, 6)$ works in $x + y = 7$, but $2(1) - 6 = -4 \neq 5$, so it does not solve $x + y = 7$, $2x - y = 5$.

2 Substitution

USE Isolate a variable with coefficient 1 or -1 . Put it into the **other** equation, in brackets. Solve, then substitute back for the second variable.

EXAMPLE $y = 2x - 1$, $3x + 2y = 12$:
 $3x + 2(2x - 1) = 12$, $7x = 14$, so $x = 2$,
 $y = 3$.

TRAP $x = y + 3$ into $2x + y = 12$ is
 $2(y + 3) + y = 12$, so $y = 2$. Without
brackets, $2y + 3 + y = 12$ gives $y = 3$.

3 Elimination

USE Multiply **every** term, constants too, until one variable has matching coefficients. Opposite signs: add. Same signs: subtract.

EXAMPLE $3x + 2y = 16$, $x - y = 2$: $\times 2$
gives $2x - 2y = 4$; add: $5x = 20$, so
 $x = 4$, $y = 2$.

TIP Check the answer in **both** original equations.

TRAP $\times 2$ on $x - y = 2$ gives
 $2x - 2y = 4$, not $2x - 2y = 2$: the
constant doubles too.

TRAP $+3y$ and $-3y$ cancel when you
add; subtracting gives $6y$.

4 Number of solutions

USE None: an **inconsistent system**.
Infinitely many: a **dependent system**. A
false result like $0 = 6$: none.

Both lines in $y = mx + b$ form:

different slopes **one: they cross**

same slope,
different b **none: parallel**

same slope, same
 b **infinitely many:
same line**

TRAP $y = 2x + 3$ and $y = -x + 3$ share
 $b = 3$, yet they cross at $(0, 3)$.

TRAP Both variables cancel and you get
 $0 = 0$: infinitely many solutions, not none.

5 Equivalent systems

USE Multiplying an equation by a non-zero
number, or adding the two equations, gives
equivalent systems: they have the same
solution.

EXAMPLE $x + y = 5$, $x - y = 1$ and
 $2x + 2y = 10$, $2x = 6$ both have solution
 $(3, 2)$.

6 Setting up a problem

USE Say what each variable counts. Write
one equation per fact, often one for items
and one for value. Answer in a sentence.

EXAMPLE 50 tickets, \$480, adults \$12,
students \$8: $a + s = 50$, $12a + 8s = 480$,
so $a = 20$, $s = 30$.

TRAP "4 times as many cars as buses" is
 $c = 4b$, not $4c = b$: try $b = 10$.

TRAP A count like -2.5 tickets means a
wrong setup or an impossible situation.

7 Distance and mixture

distance = speed $\times$ time

USE With a current c : boat speed $b + c$
downstream, $b - c$ upstream. Mixtures: one
equation for the volumes, one for the pure
part (percent $\times$ volume).

EXAMPLE 36 km downstream in 2 h,
back in 3 h: $b + c = 18$, $b - c = 12$, so
 $b = 15$ km/h, $c = 3$ km/h.

TRAP Speed is distance $\div$ time: $36 \div$
 $2 = 18$ km/h, not $2 \div 36$.