

Quadratic Equations

Ontario MPM2D • Quadratic Relations: solving by factoring, completing the square and the quadratic formula

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Worksheet, answers &
this cheat sheet



MOST TESTED Solve $3x^2 + 5x - 1 = 0$. Give exact roots and roots to two decimal places. (card 4)

Which method?

It factors with small integers

Factor

$a = 1$ and b is even

Complete the square

It doesn't factor

Quadratic formula

1 Solving by factoring

$$ab = 0 \Rightarrow a = 0 \text{ or } b = 0$$

USE Move every term to one side so it equals 0, factor, then set each factor equal to 0.

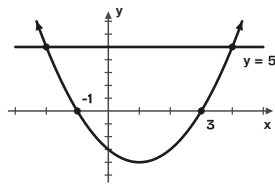
EXAMPLE $3x^2 - 7x - 6 = 0$:
 $(3x + 2)(x - 3) = 0$, so $x = -2/3$ or $x = 3$.

TRAP $(x - 2)(x + 4) = 7$ is not $x - 2 = 7$: expand to $x^2 + 2x - 15 = 0$, so $x = 3$ or -5 .

TRAP Don't divide $x^2 = 5x$ by x : you lose $x = 0$. Write $x(x - 5) = 0$.

2 Roots on a graph

USE The roots of $ax^2 + bx + c = 0$ are the x -intercepts of $y = ax^2 + bx + c$. To solve $ax^2 + bx + c = k$, read where the graph meets $y = k$.



EXAMPLE $x^2 - 2x - 3 = 5$: the graph meets $y = 5$ at $x = -2$ and $x = 4$.

TRAP $y = x^2 - 2x - 3$ meets the y -axis at -3 . That is the y -intercept, not a root.

3 Completing the square

USE Rewrite as $a(x - h)^2 + k = 0$, isolate the square, then take $\pm$ the square root. A negative on the other side: no real roots.

EXAMPLE $x^2 + 6x + 2 = 0$:
 $(x + 3)^2 = 7$, so $x = -3 \pm \sqrt{7}$.

TRAP Keep the $\pm$: $(x - 1)^2 = 9$ gives $x = 4$ or $x = -2$.

TRAP $(x - 2)^2 = -3$ has no real roots; it does not give $x = 2 \pm \sqrt{3}$.

4 The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

USE First write $ax^2 + bx + c = 0$. Simplify the radical; round only at the end.

EXAMPLE $x^2 - 2x - 4 = 0$:
 $x = (2 \pm \sqrt{20})/2 = 1 \pm \sqrt{5}$, so $x \approx 3.24$ or -1.24 .

TRAP $2x^2 - 3x - 4 = 0$: $-b = +3$ and $b^2 - 4ac = 9 + 32 = 41$, not $9 - 32$.

TRAP $2a$ divides the whole top:
 $(2 \pm \sqrt{20})/2 = 1 \pm \sqrt{5}$, not $2 \pm \sqrt{5}$.

5 The discriminant

$$D = b^2 - 4ac$$

$D > 0$ **two real roots**

$D = 0$ **one real root**

$D < 0$ **no real roots**

TRAP $D = 0$ means **one** root, not a root equal to 0: $x^2 - 6x + 9 = 0$ has $x = 3$.

TRAP $x^2 + kx + 16 = 0$ has one root when $k^2 = 64$: $k = 8$ or $k = -8$.

6 Word problems

USE Set up, rearrange to $= 0$, solve, then test each root: no negative lengths, times or counts.

EXAMPLE A path inside a 10 m by 6 m garden leaves 32 m²: $(10 - 2x)(6 - 2x) = 32$ gives $x = 1$ or 7 ; only $x = 1$ m fits.

TRAP Positive is not enough: $x = 7$ gives a width of $6 - 2(7) = -8$.

TRAP $D < 0$ for a height of 10 m: the ball never gets that high.