

# Solving Quadratic Equations Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

**Recommended:** K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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## QUICK REFERENCE

**Zero product property:** if  $ab = 0$ , then  $a = 0$  or  $b = 0$ . It works only when one side is 0, so move every term to one side first.

**Quadratic formula:** for  $ax^2 + bx + c = 0$ ,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

The  $2a$  divides the whole numerator.

In a word problem, test every root against the situation. A root can be a negative length, a width bigger than the space, or a moment after the model stops applying.

Show your work on separate paper.

## Core Skills (K)

Factoring · Completing the Square · Quadratic Formula · Discriminant

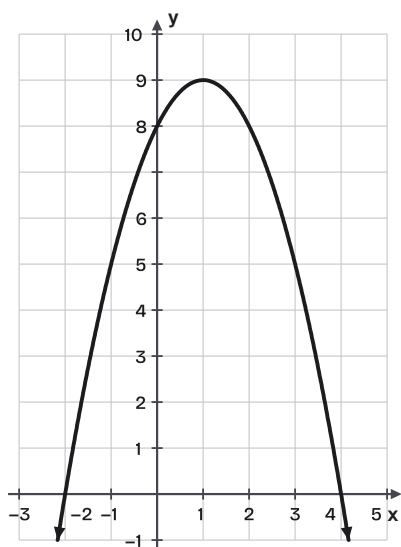
### K1 Zero Product Property

Solve each equation.

- a)  $(x - 4)(3x + 2) = 0$
- b)  $x^2 = 6x$
- c)  $(x - 3)(x + 1) = 5$

### K2 Roots on the Graph

The graph of  $y = -x^2 + 2x + 8$  is shown. Each grid square is 1 unit.



- a) Use the graph to solve  $-x^2 + 2x + 8 = 0$ .
- b) Use the graph to solve  $-x^2 + 2x + 8 = 5$ .
- c) Determine the number of solutions of  $-x^2 + 2x + 8 = k$  when  $k = 9$  and when  $k = 10$ .

**Completing the square:** for  $x^2 + bx$ , add and subtract  $\left(\frac{b}{2}\right)^2$ . If  $a \neq 1$ , factor  $a$  out of the  $x$ -terms first: what you add inside the bracket is multiplied by  $a$ .

**Discriminant**  $D = b^2 - 4ac$ :  $D > 0$  gives two real roots,  $D = 0$  one,  $D < 0$  none.

### K3 Completing the Square

Solve by completing the square. Give exact answers, or state that there are no real roots.

- a)  $x^2 + 10x + 18 = 0$
- b)  $x^2 - 6x + 14 = 0$
- c)  $2x^2 - 12x + 7 = 0$

### K4 Quadratic Formula

Solve using the quadratic formula. Give exact answers in simplest form.

- a)  $x^2 - 4x - 6 = 0$
- b)  $3x^2 + 2x - 2 = 0$
- c)  $2x^2 = 5x + 1$ . Also give the roots to two decimal places.

### K5 The Discriminant

Without solving, find the value of the discriminant and state the number of real roots.

- a)  $x^2 - 6x + 9 = 0$
- b)  $3x^2 - 4x + 5 = 0$
- c)  $-2x^2 + 3x + 4 = 0$

## Real World (A)

Projectiles · Area · Profit

### A1 Gym Ceiling

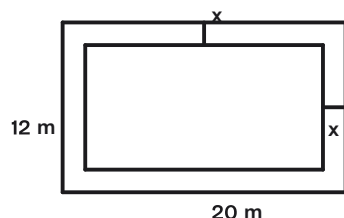
In a school gym in Sudbury, a student bumps a volleyball upward. Its height, in metres, after  $t$  seconds is modelled by  $h(t) = -5t^2 + 12t + 2$ .

- a) If nothing were in the way, when would the ball hit the floor? Round to the nearest hundredth of a second.

- b) Use the discriminant to determine whether the ball reaches a height of 10 m.
- c) The gym ceiling is 8 m high. Solve  $h(t) = 8$ . What does this mean for your answer to (a)?

### A2 Garden Path

A rectangular community garden in Guelph measures 20 m by 12 m. A gravel path of uniform width  $x$  metres runs along all four sides, inside the fence, as shown (not to scale). The planting bed in the middle must have an area of  $128 \text{ m}^2$ .



- a) Write an equation for the area of the planting bed and rearrange it to the form  $ax^2 + bx + c = 0$ .
- b) Solve the equation.
- c) Which solution fits the garden? State the path width and the dimensions of the planting bed.

### A3 Mug Market

A student sells handmade mugs at a weekend market in Kingston. Each mug costs \$10 to make. The table shows how many mugs,  $n$ , sell per day at a price of  $x$  dollars per mug.

|     |     |     |    |    |
|-----|-----|-----|----|----|
| $x$ | 15  | 20  | 25 | 30 |
| $n$ | 125 | 100 | 75 | 50 |

- a) Write an equation for  $n$  in terms of  $x$ . Then show that the daily profit, in dollars, is  $P(x) = -5x^2 + 250x - 2000$ .
- b) Find the prices that give a daily profit of \$1000, and the number of mugs sold at each price.
- c) The formula gives  $P(50) = -2000$ . What does the model predict at a price of \$50, and why can't that happen?

## Think It Through (T)

Error Analysis · Building Equations

### T1 Spot the Flaw

A student solves  $3x^2 + 12x - 5 = 0$  by completing the square.

Line 1:  $3(x^2 + 4x) - 5 = 0$

Line 2:  $3(x^2 + 4x + 4) - 5 - 4 = 0$

Line 3:  $3(x + 2)^2 = 9$

Line 4:  $(x + 2)^2 = 3$

Line 5:  $x + 2 = \sqrt{3}$ , so  $x = -2 + \sqrt{3}$ .

- a) Substitute the student's answer into  $3x^2 + 12x - 5$ . Is it a root?
- b) Two lines contain an error. Identify each one and correct it.
- c) Solve the equation correctly. Give the roots in exact form and to two decimal places.

### T2 Build It Backwards

- a) Write a quadratic equation in standard form, with integer coefficients, whose roots are  $\frac{3}{2}$  and  $-4$ .
- b) Find every value of  $k$  for which  $x^2 + kx + 9 = 0$  has exactly one real root.
- c) Find the values of  $k$  for which  $2x^2 - 4x + k = 0$  has no real roots.

## Boss Level

Optional challenge

### A Quadratic in Disguise

Consider the equation  $(x^2 - 3x)^2 - 2(x^2 - 3x) - 8 = 0$ .

- a) Let  $u = x^2 - 3x$ . Rewrite the equation in terms of  $u$  and solve for  $u$ .
- b) Find all real solutions  $x$  of the original equation.
- c) Find the value of  $k$  for which  $(x^2 - 3x)^2 - 2(x^2 - 3x) = k$  has exactly three real solutions.



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