

Quadratic Relations

Ontario MPM2D • Quadratic Relations: finite differences, parabolas, factored form and models

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Worksheet, answers &
this cheat sheet



MOST TESTED For $y = 2(x - 1)(x + 5)$, state the zeros, the axis of symmetry and the vertex.
(card 3)

Where to start?

A table of values

Differences

An equation in
factored form

Zeros, then axis

A greatest area or
revenue

Vertex y -value

1 Finite differences

USE Difference = next – previous; keep the signs. Second difference = $2a$: positive opens up, negative opens down.

With x going up by 1:

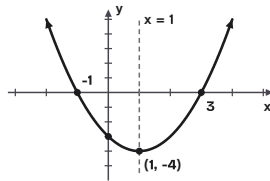
first differences constant	linear
second differences constant, not 0	quadratic
neither constant	neither

TRAP y : 5, 8, 9, 8, 5 has first differences 3, 1, -1, -3, not 3, 1, 1, 3.

TRAP Second difference -4 means $2a = -4$, so $a = -2$, not -4.

2 Parts of a parabola

USE Vertex (h, k) ; axis of symmetry $x = h$; the **maximum** or **minimum** value is k . **Zeros**: where $y = 0$.



TRAP Vertex $(1, -4)$: the minimum value is -4 and the axis is $x = 1$, not $y = 1$ or the point.

TRAP y is equal at $x = 0$ and $x = 1$, so the axis is $x = 0.5$: the vertex need not be in the table.

3 Factored form

$$y = a(x - r)(x - s)$$

$$\text{axis } x = \frac{r + s}{2}$$

USE Zeros r and s . $a > 0$ opens up; $a < 0$ opens down. Vertex: substitute the axis value.

EXAMPLE $y = 2(x - 1)(x + 5)$: zeros 1, -5; axis $x = -2$; $y = 2(-3)(3) = -18$, so vertex $(-2, -18)$.

TRAP $(x + 5)$ gives the zero -5, not 5: solve $x + 5 = 0$.

TRAP The lone factor x in $y = 3x(x - 4)$ is a zero too: $x = 0$.

4 Finding a ; best fit

USE From the zeros, write $y = a(x - r)(x - s)$; substitute one more point to solve for a . A curve of best fit follows a scatter plot's trend.

EXAMPLE Zeros -1 and 4, through $(0, 8)$: $8 = a(1)(-4)$, so $a = -2$.

TRAP Solve for a : it is not 1, the y -intercept or the maximum height.

TRAP Interpolate inside the data; extrapolating past it (after the ball lands) can give nonsense.

5 Expanding

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

USE Multiply the two brackets first (every term by every term, or an area model), then multiply by the constant in front.

EXAMPLE $-2(x - 3)(x + 1) = -2(x^2 - 2x - 3) = -2x^2 + 4x + 6$.

TRAP $(3x - 2)^2 = 9x^2 - 12x + 4$, not $9x^2 + 4$; write it as $(3x - 2)(3x - 2)$.

6 Maximum and minimum

USE Write the quantity in factored form. Zeros, then the axis, then substitute: the maximum is the y -value. Answer the question asked.

EXAMPLE 40 m of fence, three sides, a wall on the fourth: $A = w(40 - 2w)$, zeros 0 and 20, so $w = 10$ m and $A = 200 \text{ m}^2$.

TRAP A vertex at $x = -10$ price increases means **lowering** the price by \$10; don't flip the sign.

7 Zeros in context

USE A zero is where the quantity is 0: the ball is on the ground, or no revenue. Keep only values that make sense, like $t \geq 0$.

EXAMPLE $h = -5t(t - 4)$: zeros $t = 0$ (kick) and $t = 4$ s (landing); maximum $h(2) = 20$ m.