

Introduction to Quadratic Relations Review

Try each problem first. You don't need to finish everything.

Core path ~55 min · Full sheet ~1 h 45 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding · A = Application · T = Thinking · Boss = Challenge



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QUICK REFERENCE

Second differences: when x goes up by 1 each time, constant second differences (not 0) mean the relation is quadratic, and the second difference equals $2a$. Positive: opens up (minimum). Negative: opens down (maximum).

Factored form $y = a(x - r)(x - s)$: the zeros are r and s , so $(x + 5)$ gives the zero $x = -5$. If $a > 0$ the parabola opens up; if $a < 0$ it opens down.

Expanding: $(x + b)^2 = x^2 + 2bx + b^2$, never $x^2 + b^2$. Multiply the two brackets first, then multiply by a .

Show your work on separate paper.

Core Skills (K)

Second Differences · Properties · Factored Form · Expanding

K1 Linear, Quadratic or Neither?

For each table, determine the first and second differences. Then state whether the relation is linear, quadratic or neither. If it is quadratic, state the direction of opening.

a) Table A

x	0	1	2	3	4
y	6	9	10	9	6

b) Table B

x	-2	-1	0	1	2
y	-5	-2	1	4	7

c) Table C

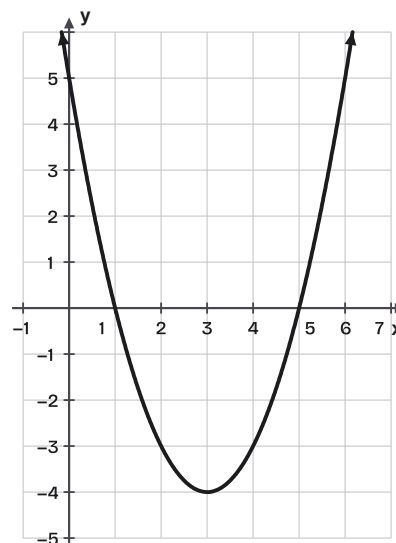
x	0	1	2	3	4
y	1	3	9	27	81

Find differences as **next minus previous** and keep the signs. Turning the differences positive hides a quadratic pattern.

Vertex: the axis of symmetry is halfway between the zeros, $x = \frac{r + s}{2}$. Substitute that x into the equation to get the maximum or minimum value.

K2 Read the Parabola

Use the graph below. Each grid square is 1 unit.



- State the coordinates of the vertex and the equation of the axis of symmetry.
- State the zeros and the y -intercept.
- State whether the relation has a maximum or a minimum value, and state that value.

K3 Symmetry in a Table

The table shows a quadratic relation.

x	-3	-2	-1	0	1	2
y	-4	2	6	8	8	6

- Determine the equation of the axis of symmetry.

- b) Use symmetry to determine y when $x = 3$ and when $x = 4$.
- c) The relation is $y = -x^2 + x + 8$. Determine the coordinates of the vertex and the maximum value.

K4 Factored Form

- a) For $y = 2(x - 3)(x + 5)$, state the zeros, the direction of opening and the y -intercept.
- b) Determine the equation of the axis of symmetry and the vertex of $y = 2(x - 3)(x + 5)$.
- c) For $y = -x(x - 6)$, state the zeros and determine the vertex.

K5 Expand to Standard Form

- a) Complete the area model, then expand and simplify $(x + 6)(x - 2)$.

$\times$	x	-2
x		
6		

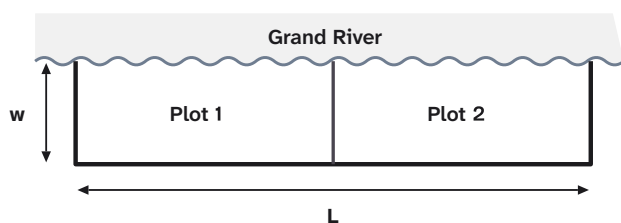
- b) Expand and simplify $(2x - 3)^2$.
- c) Write $y = -3(x - 4)(x + 1)$ in standard form.

Real World (A)

Optimization · Revenue · Modelling Data

A1 Garden by the River

A community garden along the Grand River is fenced into two equal rectangular plots using 60 m of fencing, as shown. The side along the river needs no fence. Let w be the width and L the total length along the river, both in metres.



- a) Write L in terms of w . Then write the total area $A(w)$ in factored form.
- b) Determine the dimensions that give the maximum total area, and state that area.
- c) The flat stretch of riverbank is only 24 m long, so $L \leq 24$. Determine the largest possible total area now.

A2 Climbing Gym Passes

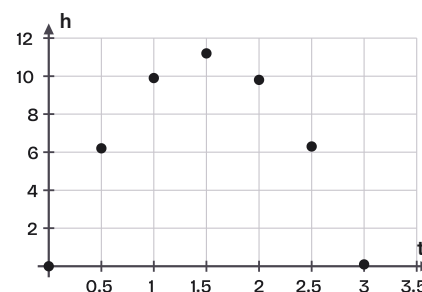
A climbing gym in Ottawa sells 100 day passes a week at \$40 each. For every \$1 increase in price, it sells 5 fewer passes; for every \$1 decrease, 5 more. Let x be the number of \$1 increases, so a negative x is a decrease.

- a) Write the weekly revenue $R(x)$ in factored form and state its zeros.
- b) What does each zero mean for the gym?
- c) Determine the vertex. What price maximizes weekly revenue, and what is the maximum revenue?

A3 Tracking a Kick

A drone films a soccer ball kicked from the ground. The table and scatter plot show its height h , in metres, t seconds after the kick. The ball lands at about $t = 3$ s.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
h (m)	0	6.2	9.9	11.2	9.8	6.3	0.1



- a) Use the zeros and the point $(1.5, 11.2)$ to determine a model $h = at(t - 3)$. Round a to the nearest whole number.
- b) Use your model to determine the height at $t = 0.8$ s. At what other time is the ball at that height?
- c) Use your model to predict the height at $t = 3.4$ s. Is the prediction realistic?

Think It Through (T)

Error Analysis · Building Equations

T1 Spot the Flaw

A student analyzes the table below.

"The second differences are all -4 , so $a = -4$ and the parabola opens down. The y -intercept is 3, so the vertex is $(0, 3)$."

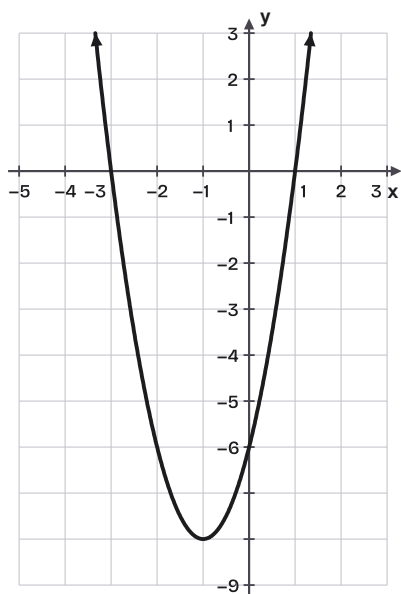
x	0	1	2	3	4
y	3	5	3	-3	-13

- a) Determine the second differences of $y = -4x^2$ for $x = 0, 1, 2, 3$. What value of a gives a second difference of -4 ?
- b) Use the table to determine the vertex. What is the point $(0, 3)$?
- c) Use only the values at $x = 0, 2$ and 4 (steps of 2). Determine the second difference. Does it still equal $2a$?

T2 Build It Backwards

Write an equation in factored form for each parabola.

- a) Zeros -2 and 6 , and y -intercept 6 .
- b) Vertex $(3, 27)$, and one zero at $x = 0$.
- c) The parabola graphed below. Each grid square is 1 unit.



Boss Level

Optional challenge

Chords of a Parabola

Consider the parabola $y = (x - 2)(x - 8)$ and the horizontal line $y = k$.

- a) The line $y = k$ meets the parabola at two points that are 10 units apart. Find k .
- b) For which value of k does the line meet the parabola at exactly one point?
- c) For which value of a does the line $y = 16$ meet $y = a(x - 2)(x - 8)$ at two points that are 4 units apart?



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