

Quadratics in Vertex Form

Ontario MPM2D • Quadratic Relations: transformations, vertex form and completing the square

vertexmath.ca
Worksheet, answers &
this cheat sheet



MOST TESTED Write $y = 2x^2 - 12x + 13$ in vertex form. State the vertex and the minimum value. (card 5)

Which form?

You need the zeros

Factored form

You need the max or min

Vertex form

You need the starting value

Standard form

1 The parameter a in $y = ax^2$

$$|a| > 1$$

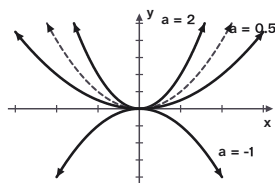
stretch: narrower

$$0 < |a| < 1$$

compression: wider

$$a < 0$$

reflection: opens down



TRAP $y = 2x^2$ at $x = 3$: $2(3)^2 = 18$, not $(2 \cdot 3)^2 = 36$. Square first.

TRAP $y = -0.5x^2$ is a vertical compression, **wider** than $y = x^2$: the sign only flips it.

2 Vertex form

$$y = a(x - h)^2 + k$$

USE Vertex (h, k) , axis $x = h$. Shift right h , up k . $a > 0$: minimum k ; $a < 0$: maximum k .

EXAMPLE $y = -2(x + 3)^2 + 5$: vertex $(-3, 5)$, maximum 5; y -intercept $-2(9) + 5 = -13$.

TRAP $y = (x + 4)^2 - 1$ has vertex $(-4, -1)$: only h changes sign, k keeps its sign.

TRAP Keep a for the y -intercept: $y = -3(x - 2)^2 + 1$ gives $-3(4) + 1 = -11$, not 5.

3 Finding the equation

USE Put the vertex in for h and k , a second point in for x and y , then solve for a .

EXAMPLE Vertex $(1, -3)$, through $(3, 5)$: $5 = a(3 - 1)^2 - 3$, so $4a = 8$ and $a = 2$.

TRAP Square the bracket: $5 = a(2)^2 - 3$ gives $a = 2$. Without the square, you get $a = 4$.

4 Models

USE The maximum or minimum is k , reached at $x = h$. Equal heights sit the same distance either side of $x = h$.

EXAMPLE $h = -5(t - 2)^2 + 21$: maximum 21 m at $t = 2$ s; the height at $t = 1$ equals the height at $t = 3$.

TRAP Far from the vertex a revenue model can go negative. Use a model only where it makes sense.

5 Completing the square

USE Factor a from the x^2 and x terms. Inside, add (half the x -coefficient)²; outside, subtract a times that number.

EXAMPLE $3x^2 + 12x + 7 = 3(x^2 + 4x + 4) + 7 - 12 = 3(x + 2)^2 - 5$.

TRAP Adding 4 inside $3(\dots)$ adds 12: subtract 12 outside, not 4.

TRAP Factoring out a negative flips the x -term: $-2x^2 + 8x = -2(x^2 - 4x)$.

6 Three forms

$$ax^2 + bx + c$$

y -intercept c

$$a(x - r)(x - s)$$

zeros r, s

$$a(x - h)^2 + k$$

vertex (h, k)

EXAMPLE

$x^2 - 6x + 5 = (x - 1)(x - 5)$: zeros 1 and 5, axis $x = 3$, $y(3) = -4$, so $(x - 3)^2 - 4$.

TRAP $(x - 3)^2 = x^2 - 6x + 9$, not $x^2 + 9$.