

Quadratic Functions in Vertex Form Review

Try each problem first. You don't need to finish everything.

Core path ~55 min • Full sheet ~1 h 50 min

Recommended: K1-K5, then one A and one T problem

K = Knowledge & Understanding • A = Application • T = Thinking • Boss = Challenge



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QUICK REFERENCE

Vertex form $y = a(x - h)^2 + k$: the vertex is (h, k) and the axis of symmetry is $x = h$. In $(x + 3)^2$, $h = -3$; the k keeps its sign.

Square first, then multiply: for $y = 3x^2$ at $x = 2$, $y = 3(2)^2 = 12$, not $(3 \cdot 2)^2 = 36$.

Completing the square: factor a out of the x^2 and x terms only. Whatever you add inside the bracket is multiplied by a , so subtract a times it outside.

Parameter a : $|a| > 1$ stretches the parabola (narrower), $0 < |a| < 1$ compresses it (wider), and $a < 0$ reflects it in the x -axis (opens down, maximum).

Finding a : substitute the vertex for h and k , a second point for x and y , then solve. Remember to square $(x - h)$.

Show your work on separate paper.

Core Skills (K)

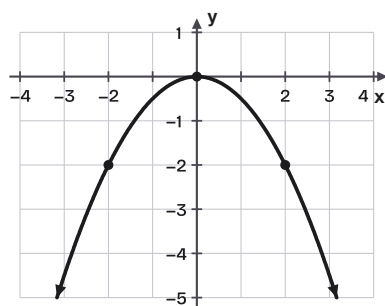
Stretches & Reflections • Translations • Vertex & a Point • Completing the Square • Three Forms

K1 The Role of a

- a) Complete the table of values for $y = 3x^2$.

x	-2	-1	0	1	2
y					

- b) The parabola $y = ax^2$ is graphed below. Determine a . Each grid square is 1 unit.



- c) Order from narrowest to widest: $y = -4x^2$, $y = 0.5x^2$, $y = 2x^2$, $y = -0.25x^2$.

K2 Reading the Transformations

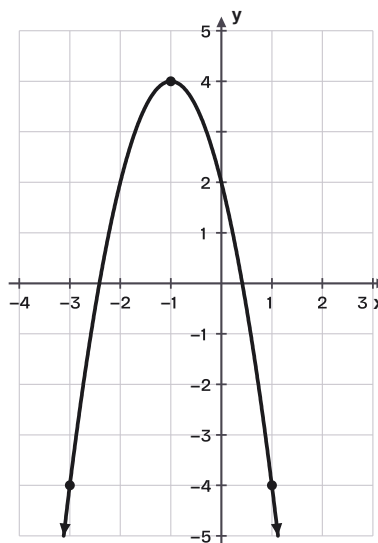
- a) State the vertex and the axis of symmetry of $y = (x + 3)^2 - 4$.
- b) For $y = -2(x - 5)^2 + 1$, state the vertex, the direction of opening and the y -intercept.
- c) The parabola $y = x^2$ is stretched vertically by a factor of 3, then translated 4 units left and 2 units down. Write its equation.

K3 Vertex and One Point

Determine the equation of each parabola in vertex form.

- a) Vertex $(2, 3)$, passing through $(4, 11)$.

- b) The parabola graphed below. Each grid square is 1 unit.



- c) Vertex $(-4, 0)$ and y -intercept 8.

K4 Completing the Square

Write each relation in vertex form by completing the square. Then state the vertex.

- a) $y = x^2 + 8x + 11$
- b) $y = 3x^2 - 18x + 25$
- c) $y = -2x^2 - 12x - 13$

K5 Three Forms, One Parabola

- a) Factor $y = x^2 - 2x - 15$ and state its zeros.
- b) Use the zeros to determine the vertex of $y = x^2 - 2x - 15$. Then write the relation in vertex form.
- c) Write $y = 2(x - 3)^2 - 8$ in standard form and in factored form.

Real World (A)

Quadratic Models · Choosing the Best Form

A1 Gym Ceiling

In a school gym, a volleyball is bumped from a height of 2 m. Its height, in metres, after t seconds is

$$h(t) = -5(t - 1)^2 + 7.$$

- State the maximum height of the ball and when it occurs.
- A teammate's hands are 2 m above the floor. When does the ball reach them on the way down?
- A harder bump from the same height is modelled by $g(t) = -5(t - 1.2)^2 + 9.2$. The gym ceiling is 9 m high. State the maximum height the model predicts, then say what really happens to the ball.

A2 Ski Hill Day Passes

A ski hill near Collingwood finds that its daily revenue from day passes is greatest, \$14 400, at a price of \$60 per pass. At a price of \$40, the daily revenue is \$12 800.

- Write a model $R(p)$ in vertex form for the daily revenue, in dollars, at a price of p dollars per pass.
- Use your model to predict the daily revenue at \$75 per pass.
- Use your model to predict the daily revenue at \$130 per pass. What does the result mean for the ski hill?

A3 Parade Arch

A parabolic arch frames the entrance to a fairground in London, Ontario. Its height, in metres, is $h(x) = -0.5x^2 + 6x - 10$, where x is the horizontal distance, in metres, from a lamp post.

- Write $h(x)$ in vertex form. State the maximum height of the arch and how far from the lamp post it occurs.
- Write $h(x)$ in factored form. How wide is the arch at ground level?
- A parade float has a flat top. It is 4.4 m wide and 6 m tall, and it drives through the centre of the arch. Does it fit?

Think It Through (T)

Error Analysis · Building Parabolas · Counterexample

T1 Spot the Flaw

A student completes the square for $y = 2x^2 + 12x + 5$:

"

$$y = 2(x^2 + 6x) + 5 = 2(x^2 + 6x + 9) + 5 - 9 = 2(x + 3)^2 - 4.$$

The minimum value is -4 , at $x = -3$."

- Evaluate the original relation at $x = -3$. What does the result tell you about the student's answer?
- Identify the error and write the correct vertex form.
- Without completing the square again, write $y = 2x^2 + 12x + 20$ in vertex form. Explain your reasoning.

T2 Build It Backwards

Use vertex form $y = a(x - h)^2 + k$ in each part.

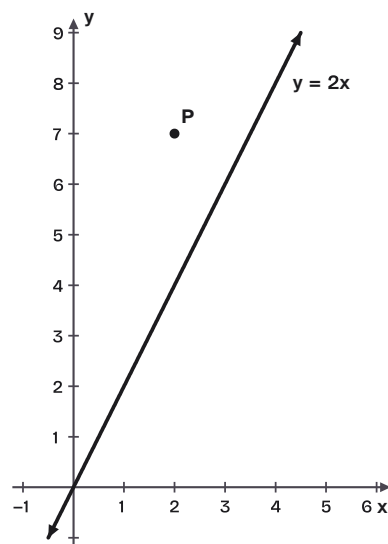
- Write an equation of a parabola with vertex $(-2, 5)$ that opens down and is narrower than $y = x^2$. Many answers are possible.
- A parabola has axis of symmetry $x = 3$ and passes through $(1, 0)$ and $(0, -10)$. Determine its equation.
- A student claims: "Every parabola that opens down crosses the x -axis." Give a counterexample. Then state the condition on k for $y = a(x - h)^2 + k$, with $a < 0$, to have two zeros.

Boss Level

Optional challenge

Closest Point on a Line

The point $P(2, 7)$ lies off the line $y = 2x$. Every point on the line has the form $Q(x, 2x)$.



- Write PQ^2 as a quadratic in x , in standard form.
- Complete the square to determine the point Q closest to P , and the shortest distance from P to the line.
- The point on the line straight across from P is $(3.5, 7)$. Determine its distance from P . Then show that PQ is perpendicular to the line.



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