

Similar Triangles and Trigonometry

Ontario MPM2D · Trigonometry: similar triangles, primary trigonometric ratios and right-triangle problems

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Worksheet, answers &
this cheat sheet



MOST TESTED From the top of a 40 m cliff, the angle of depression to a boat is 20° . How far is the boat from the base of the cliff? (card 6)

Which tool?

Two triangles with
matching angles

Proportion

Right triangle, two
sides, need the third

$$a^2 + b^2 = c^2$$

Right triangle with
an angle

SOH CAH TOA

1 Congruent or similar?

SSS, SAS or ASA **congruent: same size and shape**

two pairs of equal angles (AA) **similar**

all three side ratios equal **similar**

TRAP Adding 2 to each side of a 3-4-5 triangle gives 5-6-7: not similar, since $5/3 \neq 6/4$.

TRAP Two right triangles are similar only if one more angle matches.

2 Scale factor

$$k = \frac{\text{new side}}{\text{original side}}$$

lengths $\times k$ areas $\times k^2$

USE Match sides by vertex order: in $\triangle ABC \sim \triangle DEF$, AB goes with DE .

EXAMPLE $k = 2.5$: a side of 4 becomes 10; an area of 6 becomes $6(2.5)^2 = 37.5$.

TRAP With $k = 3$, an area of 5 cm^2 becomes 45 cm^2 , not 15.

3 Proportions and shadows

USE Write matching sides as equal ratios, in the **same order** on both sides, then cross-multiply.

EXAMPLE A 1.6 m student casts a 2 m shadow; a tree casts 15 m: $h/15 = 1.6/2$, so $h = 12 \text{ m}$.

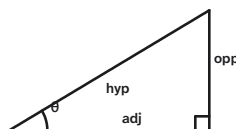
TRAP Nested triangles: $AD/AB = DE/BC$ uses the **whole** side AB , not the piece DB .

TRAP Same order: height/shadow = height/shadow, never height/shadow = shadow/height.

4 SOH CAH TOA

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

USE Stand at θ : the **opposite** side doesn't touch it; the **hypotenuse** faces the right angle. A ratio depends only on the angle, not the triangle's size.



TRAP From the other acute angle, opposite and adjacent swap: the vertical side is not always "opposite".

5 Solving a right triangle

USE Side: write the ratio, then rearrange. Angle: use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$. Third side: $a^2 + b^2 = c^2$, c the hypotenuse. Degree mode.

EXAMPLE $\cos 40^\circ = 12/x$:
 $x = 12 / \cos 40^\circ \approx 15.7$.

TRAP Unknown in the denominator: divide. $12 \cos 40^\circ \approx 9.2$ would make the hypotenuse shorter than a leg.

TRAP $\sin \theta = 0.6$ gives
 $\theta = \sin^{-1} 0.6 \approx 36.9^\circ$; $\sin^{-1}$ is not $1/\sin$.

6 Elevation and depression

USE Both are measured from the **horizontal**. Depression from the top equals elevation from the bottom. Two triangles: solve one, then carry the shared side.

EXAMPLE From 40 m up, depression 20° : $\tan 20^\circ = 40/d$, so $d = 40 / \tan 20^\circ \approx 109.9 \text{ m}$.

TRAP A ramp slope of 1 : 12 means $\tan \theta = 1/12$, so $\theta \approx 4.8^\circ$, not 12° .