

Geometry and Measurement

Ontario MTH1W · angle relationships, polygon and circle angles, the Pythagorean theorem, perimeter, area, surface area and volume

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Worksheet, answers &
this cheat sheet



MOST TESTED A 6.5 m ladder leans against a wall with its foot 2.5 m from the wall. How high up the wall does it reach? (card 4)

Which measure?

The distance
around a shape

Perimeter, in m

The space **covering**
a surface

Area, in m²

The space **inside**
a solid

Volume, in m³

1 Angle pairs

F, Z, C: only for lines **marked** parallel.

complementary **add to 90°**

supplementary (straight line) **add to 180°**

vertically opposite (X) **equal**

corresponding (F) **equal**

alternate (Z) **equal**

co-interior (C) **add to 180°**

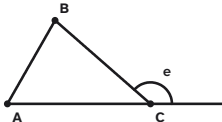
TRAP Co-interior angle 70°: the other is 110°, not 70°.

2 Triangle angles

$$\angle A + \angle B + \angle C = 180^\circ$$

$$e = \angle A + \angle B$$

USE $\angle A, \angle B$: the interior angles **not** next to e .



TRAP $e = 125^\circ$ and $\angle A = 48^\circ$:
 $\angle B = 125^\circ - 48^\circ = 77^\circ$, not 173° .

3 Polygon and circle angles

$$\text{interior sum} = (n - 2) \times 180^\circ$$

$$\text{exterior sum} = 360^\circ$$

$$\text{inscribed} = \frac{1}{2} \times \text{central}$$

USE Regular polygon: divide by n for one angle. Circle: both angles on the same arc.

TRAP Decagon: exterior angles add to 360° , not $8 \times 180^\circ$.

TRAP Central angle 110° : inscribed angle 55° , not 110° .

4 Pythagorean theorem

$$a^2 + b^2 = c^2$$

$$a^2 = c^2 - b^2$$

USE c is the **hypotenuse**: opposite the right angle, the longest side.

EXAMPLE Ladder 6.5 m, foot 2.5 m out:
 $h^2 = 6.5^2 - 2.5^2 = 36$, so $h = 6$ m up the wall.

TRAP Hypotenuse 13, leg 5: the other leg is $\sqrt{169 - 25} = 12$, not $\sqrt{194}$, longer than 13.

5 Area of polygons

USE h meets b at 90° . Composite: split, then add or subtract.

$$\text{rectangle} \quad A = lw$$

$$\text{parallelogram} \quad A = bh$$

$$\text{triangle} \quad A = \frac{bh}{2}$$

$$\text{trapezoid} \quad A = \frac{(a+b)h}{2}$$

TIP Fixed fencing: a square gives the most area. Along a river (3 sides): length = $2 \times$ width.

TRAP Parallelogram, base 10, slanted side 6, height 5: $A = 10 \times 5 = 50$, not $10 \times 6 = 60$.

6 Circles

$$C = 2\pi r = \pi d$$

$$A = \pi r^2$$

$$\text{sector: } A = \frac{\theta}{360^\circ} \times \pi r^2$$

USE Find $r = d \div 2$ first. A semicircle's perimeter is half the circumference **plus** the diameter.

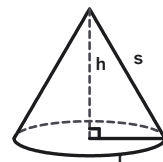
TRAP Diameter 10 cm: $A = \pi(5)^2 \approx 78.5 \text{ cm}^2$, not $\pi(10)^2 \approx 314.2 \text{ cm}^2$.

7 Surface area

$$\text{cylinder: } SA = 2\pi r^2 + 2\pi rh$$

$$\text{cone: } SA = \pi r^2 + \pi rs$$

USE Prism, pyramid: add the areas of the net. s : slant height, along the side.



TRAP Two 2 cm cubes glued: $48 - 2 \times 4 = 40 \text{ cm}^2$, not 48 cm^2 . Glued faces are hidden.

8 Volume

$$\text{prism, cylinder: } V = Bh$$

$$\text{pyramid, cone: } V = \frac{1}{3}Bh$$

USE B : base area (πr^2 for a circle). h : perpendicular height. $1 \text{ cm}^3 = 1 \text{ mL}$.

EXAMPLE Silo: cylinder $r = 2$, $h = 10$,
cone $h = 3$ on top: $40\pi + 4\pi = 44\pi \approx 138.2 \text{ m}^3$.

TRAP Cone, $r = 3$, $h = 4$, slant 5:
 $V = 1/3 \times \pi(9)(4) = 12\pi$, not 15π (slant) or 36π (no $1/3$).

9 Changing dimensions

Multiply **every** length by k :

$$\text{perimeter} \quad \times k$$

$$\text{area, surface area} \quad \times k^2$$

$$\text{volume} \quad \times k^3$$

TIP Cylinder, cone: $r \times 2$ alone gives $V \times 4$; $h \times 2$ alone gives $V \times 2$.

TRAP Cone filled to half its height:
 $(1/2)^3 = 1/8$ full, not $1/2$.