

Linear Relations

Ontario MTH1W · first differences, slope, $y = mx + b$, equations of lines, parallel and perpendicular lines, inequalities and models

vertexmath.ca
Worksheet, answers &
this cheat sheet



MOST TESTED Find the equation of the line through $(-1, 7)$ and $(3, -5)$. (card 5)

Equation of a line: first step

Two points

Slope formula

Slope and a
point

Solve for b

Parallel
or $\perp$ line

Same m or $-1/m$

1 First differences

$$\text{rate of change} = \frac{\Delta y}{\Delta x}$$

Subtract consecutive y -values:

equal steps, constant **linear**

equal steps, changing **nonlinear**

unequal x -steps **use $\Delta y \div \Delta x$**

EXAMPLE x : 0, 2, 4, 6; y : 5, 11, 17, 23:
step 2, differences 6, so $y = 3x + 5$.

TRAP x goes up by 2 and y by 6: the
rate is $6 \div 2 = 3$, not 6.

2 Direct or partial?

USE The y -intercept is the starting value.

line through $(0, 0)$ **direct variation, $y = mx$**

starts at $b \neq 0$ **partial variation, $y = mx + b$**

EXAMPLE Taxi: \$4 plus \$2 per km, so
 $C = 2d + 4$.

TRAP $C = 2d + 4$: 1 km costs \$6, but 2
km costs \$8, not \$12. Only direct
variation doubles.

3 Slope

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

risers left to right **$m > 0$**

falls left to right **$m < 0$**

horizontal, $y = k$ **$m = 0$**

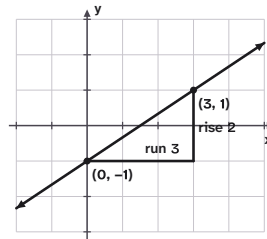
vertical, $x = h$ **undefined**

EXAMPLE $(-2, 5)$ and $(4, -1)$:
 $m = (-1 - 5)/(4 - (-2)) = -6/6 = -1$.

TRAP $(1, -3)$ to $(4, 6)$:
 $m = (6 - (-3))/(3) = 3$, not
 $(6 - 3)/3 = 1$. Bracket negatives.

4 $y = mx + b$

USE Isolate y first. Graph: plot $(0, b)$, then
rise and run by m . In a table, b is y at $x = 0$.



TRAP $y = 5 - 2x$: $m = -2$ and $b = 5$,
not $m = 5$.

5 Equation of a line

USE Find m . Put one point into $y = mx + b$;
solve for b . Check the other point.

EXAMPLE $(-1, 7)$, $(3, -5)$:
 $m = -12/4 = -3$; $7 = -3(-1) + b$, so
 $b = 4$: $y = -3x + 4$.

TRAP $m = 3$ through $(2, -1)$:
 $-1 = 6 + b$, so $b = -7$, not 2 or -1 .

TRAP $2x - 5y = 10$: $-5y = -2x + 10$;
divide **every** term: $y = 0.4x - 2$, not
 $0.4x + 10$.

6 Parallel and perpendicular

$$\text{parallel: } m_2 = m_1$$

$$\text{perpendicular: } m_2 = -\frac{1}{m_1}$$

USE Negative reciprocals: product -1 .
Horizontal $\perp$ vertical.

EXAMPLE Perpendicular to $y = 3x + 1$
through $(6, 2)$: $m = -1/3$; $2 = -2 + b$,
so $y = -x/3 + 4$.

TRAP Perpendicular to slope $2/3$: flip
and change the sign: $-3/2$, not $3/2$ or
 $-2/3$.

7 Linear inequalities

USE Test a point off the line, e.g. $(0, 0)$: true
means shade its side.

Isolate y first.

$<$ or $>$ **dashed boundary**

$\leq$ or $\geq$ **solid boundary**

$y >$ or $y \geq$ **shade above**

$y <$ or $y \leq$ **shade below**

TRAP $-2y < 4x - 6$: dividing by -2
flips the sign: $y > -2x + 3$, not
 $y < -2x + 3$.

8 Linear models

USE Slope: rate of change, with units. b :
starting value. Extrapolate only where the
situation still makes sense.

EXAMPLE A hall costs \$150 plus \$12
per guest: $C = 12g + 150$; 40 guests cost
\$630.

TRAP Tank: $V = 300 - 12t$ gives -60 L
at $t = 30$. It is empty at $t = 25$, so
 $0 \leq t \leq 25$.